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The use of global sensitivity analysis for improving processes: Applications to mineral processing

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ABSTRACT

This paper analyzes the application of global sensitivity analysis (GSA) to the improvement of processes using various case studies. First, a brief description of the methods applied is given, and several case studies are examined to show how GSA can be applied to the study to improve the processes. The case studies include the identification of processes; comparisons of the Sobol, E-FAST and Morris GSA methods; a comparison of GSA with local sensitivity analysis; an examination of the effect of uncertainty levels and the type of distribution function on the input factors; and the application of GSA to the improvement of a copper flotation circuit. We conclude that GSA can be a useful tool in the analysis, comparison, design and characterization of separation circuits. In addition, we conclude that using the stage's recoveries of each species as input factors is a suitable choice for the GSA of a flotation plant.

1. Introduction

Mineral processing comprises many unit operations, such as gravitational, magnetic and flotation stages, which are aimed at extracting valuable material from ores. Usually, the processes' operating conditions are defined to control the balance between a high recovery rate of the desired metal and a high grade value of the metal in the product outflow (Méndez, Gálvez, & Cisternas, 2009a). These processes usually include multiple stages that are interconnected (forming circuits) to maximize the recovery rate and concentrate grade. The design and analysis of these circuits, including the design and analysis of each stage, continues to be a challenging task (Ghobadi, Yahyaei, & Banisi, 2011).

A designer initially solves a synthesis problem (for any process) by trial-and-error. There are many arrangements of a concentration circuit that correspond to an acceptable trial-and-error solution; however, many of these arrangements can be incorrect, ineffective or uneconomical, which is realized when feedback on an existing process becomes available. Concentration circuits commonly evolve over time solving a number of existing problems while creating new ones (Schena & Casali, 1994).

Several methods for the design of these circuits have been presented in the literature; these methods attempt to develop a systematic procedure to replace the trial-and-error method, which is time-consuming and requires much experimentation. Among the methods developed are those that use heuristics to develop a feasible design or that improve an existing design (Connolly & Prince, 2000). However, these procedures use rules that are not always satisfied or that contradict each other and therefore do not guarantee an optimal design. Other methods use optimization or mathematical programming procedures (Cisternas, Méndez, Gálvez, & Jorquera, 2006; Ghobadi et al., 2011; Méndez, Gálvez, & Cisternas, 2009b) using a superstructure to create a set of alternatives from which an optimum design can be selected. However, the use of these methods requires training in optimization techniques because the problems are usually formulated as MINLP models for which there are no commercial codes available that ensure optimality. For the aforementioned reasons, none of the developed methodologies are widely used in industry.

The concentration stage is difficult to model, and ore characteristics vary among mining operations. Currently, there is no theoretical model that can predict the floatability of different species of a mineral and thus experimentation is necessary to develop models that can be used to design these systems. However, these experimentally based models have a limited range of application depending on the experimental conditions and the number of experiments used. The compositions and mineralogical species vary among mining operations, which in turn affects the floatability

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behavior and undermines the model validity as well as the operational parameters that are limited based on design ranges. Thus, there are at least two sources of uncertainty: the model and the ore characteristics.

Sensitivity analysis (SA) can be employed to address uncertainties in the model and application scenarios, thereby facilitating the evaluation of process structures and operational behaviors. Lucay, Mellado, Cisternas, and Gálvez (2012) applied a local SA to analyze and design separation circuits. The authors studied the effect of each stage on the general circuit by identifying the relation between the recovery rate of each stage and the global recovery rate of the circuit. Mellado, Gálvez, Cisternas, and Ordoñez (2012) applied local SA to heap leaching to validate the analytical model as well. However, local SA only considers the neighborhood of the input variation, and the effect of each input parameter is measured by keeping all the other input parameters at their nominal values. Global sensitivity analysis (GSA) can overcome these limitations and has other advantages (Saltelli, Tarantola, & Campolongo, 2000). Fesanghary, Damangir, and Soleimani (2009) studied the use of GSA and a harmony search algorithm for the design optimization of shell and tube heat exchangers (STHXs) from the economic viewpoint. GSA was used to reduce the size of the optimization problem; non-influential geometrical parameters that have the least effect on total cost of STHXs are identified and are ignored in the optimization calculation. Later, Schwier, Hartge, Werther, and Gruhn (2010) used GSA in the flow sheet simulation of solid processes, which allowed for the examination and quantification of the influences of given parameters on specific target criteria. GSA was used to decrease the effort required for the parameter estimation in a given process simulation by focusing the effort on the most influential parameters.

This work attempts to show how a GSA can be used in the analysis, design and retrofit of concentration circuits and the equipment that compose it. This work is expected to complement current design techniques, such as trial-and-error methods, heuristics or optimization. Various methodologies of GSA are analyzed and the effect of the nature of the uncertainty of the input factors is studied.

2. Global sensitivity analysis

According to Saltelli et al. (2008), the SA can be defined as "the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input". These techniques have been widely used in various engineering areas and are of great importance in determining the most significant variables in a model. The general objectives of GSA are (Reuter & Liebscher, 2008): (a) The identification of the significant and insignificant factors and the possible reduction of the dimensions (number of design variables) of an optimization problem. (b) The improvement in the understanding of the model behavior (highlighting interactions among factors and finding combinations of factors that result in high or low values for the model output). SA can be classified as: (a) Local sensitivity analysis (LSA) or differential sensitivity analysis, which is represented by the first partial derivative of a model under evaluation, producing a coefficient that describes the rate of change between the model output and one model factor while all the other factors remain constant. Its main advantage is its easy implementation and evaluation; however, it can only assess a single factor at a time (Hamby, 1994). (b) GSA (Morris Max, 1991; Reuter & Liebscher, 2008; Saltelli, Tarantola, Campolongo, & Ratto, 2007; Saltelli et al., 2008; Storlie & Helton, 2008), which for some, is identical to SA (Reuter & Liebscher, 2008) and corresponds to the evaluation of an output model when all the model factor are simultaneously evaluated, being mainly resolved by numerical methods (Monte Carlo method, Quiasi Monte Carlo and Latin Hypercube). This methodology has the advantage of simultaneously assessing all factors; however, it requires a large amount of data for which the model is evaluated using, and the mathematical techniques are more complex. GSA methods can be classified into three groups (Confalonieri, Bellocchi, Bregaglio, Donatelli, & Acutis, 2010): (1) Regression methods: The standardized regression coefficients are based on a linear regression of the output on the input vector. Linear regression is the most commonly used, but there are other techniques that are also in this group (Storlie & Helton, 2008). (2) Screening methods: This refers to the method developed by Morris with significant modification as given by Campolongo, Cariboni, and Saltelli (2007), being described in detail in Section 2.2. (3) Variance-based methods (Reuter & Liebscher, 2008; Saltelli et al., 2007, 2008; SimLab, 2008): This is a GSA method in which the variance of the model output can be decomposed into terms of increasing dimension, called partial variances, that represent the contribution of the inputs (i.e., single inputs, pairs of inputs, etc.) to the overall uncertainty of the model output. This method enables the simultaneous exploration of the space of the uncertain inputs, which is usually carried out via Monte Carlo sampling. Statistical estimators of partial variances are available to quantify the sensitivities of all the inputs and of groups of inputs through multi-dimensional integrals. The computational cost, in terms of model simulations, of estimating the sensitivities of higher-order interactions between inputs can be very high. To preclude a high computation cost, Homma and Saltelli (1996) introduced the concept of a total sensitivity index. The total sensitivity index indicates the overall effect of a given input by considering all the possible interactions of the respective input with all the other inputs. Examples of techniques in this group include the analysis of variance (ANOVA), Fourier amplitude sensitivity test (FAST), extended Fourier amplitude sensitivity test (E-FAST), Sobol' method and the high-dimensional model representation (HDMR).

2.1. FAST and Sobol' method

This method is based on the partitioning of the total variance of the model output $V(Y)$, considering that the model has the form, $Y = f(x_1, x_2, \ldots, x_n)$, where Y is a scalar and x_i is a model factor, using the following equation (Confalonieri et al., 2010):

$$V(Y) = \sum_{i=1}^{n} D_i + \sum_{i \le j \le n}^{n} D_{ij} + \cdots + \sum_{i \le \ldots n}^{n} D_{i \ldots n}, \tag{1}$$

where D_i represents the first order effect for each factor $x_i(D_i = V[E(Y|x_i)])$ and $D_{ij}(D_{ij} = V[E(Y|x_i, x_j)] - D_i - D_j)$ on D_{ij} as the interactions among n factors. The variance of the conditional expectation ($V[E(Y|x_i)]$) is sometimes called the main effect and is used as an indicator of the significance of x_i. The variance-based methods (the FAST and Sobol' methods) allow the calculation of two indices, i.e., the first-order-effect sensitivity index corresponding to a single factor (x_i)

$$S_i = \frac{V[E(Y|x_i)]}{V(Y)} \tag{2}$$

and the total sensitivity index corresponding to a single factor (index i) and the interaction of additional factors that involve the index i and at least one index $j \neq i$ from 1 to n

$$ST_i = \sum_{i} S_i + \sum_{j \neq i} S_{ij} + \cdots + S_{1,\ldots,n} \tag{3}$$

The first order sensitivity index measures only the main effect contribution of each input factor on the output variance. It does not take into account the interactions among factors. Two factors are said to interact if their total effect on the output is not equal