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Investigation on the dynamics of air-gun array bubbles based on the dual fast multipole boundary element method

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ABSTRACT

The air-gun array has important applications in ocean engineering. In this work, new dynamic behaviors of the air-gun array bubbles were investigated. The new adaptive fast multipole boundary element method (FMBEM) was implemented to accelerate the calculation of large scale moving boundary problems. The dual boundary integral equation (BIE) with an optimal linear combination of the regular BIE and the gradient BIE was used to facilitate a faster convergent FMBEM. It has been verified that the accuracy of the present numerical model was satisfactory for the dynamic simulation and the overall computational cost was scaled to $O\left(N^{1.1}\right)$ with N being the number of unknowns, which surpassed the conventional BEM largely. The behavior of air-gun array bubbles was studied for different distribution forms of the air-guns with the gravity effect considered. By increasing the dimensions of the bubble array, it can be seen that the "shield effect" of outer bubbles on inner bubbles was more evident. Pressure gradient was the dominant factor that determined the re-entrant jet's direction. The period of bubble array was longer if array scale increased. The distance between bubbles versus the intensity of interaction effect has been studied. In addition, the evolutions and the movements of the bubbles are quite different in this kind of large scale problem.

1. Introduction

The exploration of seabed mineral deposits and submarine morphology is an important work in ocean engineering nowadays. When the high pressure bubble oscillates in water, the induced pressure waveform can be utilized as the tool for this task (de Graaf et al., 2014b; Rayleigh, 1917; Safar, 1976). In the early days, the bubble created by underwater explosion was used to conduct the detection (Cole, 1965; Keller and Kolodner, 1956). Since the mid-1960s when the air-gun was invented, bubble triggered by air-gun has played the leading role in ocean survey. Compared with the detonating bubble, the air-gun bubble is environmentally friendly, economical and has stable repeatability (Johnson, 1994).

When the air-gun was firstly applied in the ocean engineering, single gun with high gas capacity was adopted. Later on, the air-gun array came on the scene. In contrast with the single air-gun, whose pressure signature is not so stable, the air-gun array has many advantages and is now widely utilized for the offshore geophysical prospecting. In shallow sea, the air-gun array may improve the detecting resolution. In deep water, the frequency of the signature created by air-gun array is much lower, which is beneficial for the oil/gas exploration (Giles, 1968). The multiple bubbles produced by the air-gun reveal a variety of highly non-linear dynamic behaviors, which have attracted the interests of many researchers for decades (de Graaf et al., 2014a; Johnson, 1994; Ziolkowski, 1998).

There are several approaches to investigate the bubble dynamics. The experimental method, as the most straightforward and convincing way, is supposed to be mentioned firstly. Since the real field measurement may consume large amounts of material and human resources, scaled experiments conducted in small charge weight are often used to study damage characteristics of structure (Ming et al., 2016). In last decades, the electric spark-generated bubble devices along with the high-speed photography have been developed rapidly. Various bubble-boundary interaction phenomena have been observed and analyzed. (Chahine and Bovis, 1980; Gibson, 1968; Gibson and Blake, 1980, 1982; Wang et al., 2015) However, there exists a drawback, that the spark-generated bubble usually has the radius of $O(1)$ cm while the air-gun bubble's is of $O(1)$ dm. Which means that the obvious buoyancy effect on the latter is no more a dominated force on the former. This difference makes the spark-generated experiment just remains referential significance for studying the air-gun bubble. It is worth mentioning that the above drawback was overcome to some

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extent since the experimental study on the bubble dynamics subjected to buoyancy has been carried out by Zhang et al. (2015a, 2015b). But with regard to the experimental investigation on the bubble array, there is a long way to go since the control of the in-phase and out-of-phase of the bubbles is still in difficulties.

Theoretical analysis is another means of studying bubble dynamics with even longer history. Throughout the last century, numerous analytical models for bubble dynamics were proposed. Among them, Plesset (1949) and Rayleigh (1917) are the celebrated pioneers. They first provided the description of the dynamics of a spherical bubble in an infinite fluid domain. In respect of the air-gun bubble, Ziolkowski (1970) published the first air-gun bubble model investigating the oscillation of a single high pressure bubble in a compressible liquid, which was extended later to Ziolkowski's air-gun array model (Ziolkowski et al., 1982). Later on, the thermodynamic state equation was introduced to govern the inner gas of the bubble by Landrø and Sollie (1992) and Langhammer and Landrø (1993). In all the models mentioned above, the major deficiency is that the bubble was assumed to vibrate spherically. Actually, even for a single air-gun bubble, the asymmetrical deformation will take place in its shrinking phase due to the buoyancy effect, not to mention the interactions between bubbles generated by the air-gun array. These situations were not precisely considered in the theoretical models. However, the problems can be solved by numerical methods.

Numerical simulations have flourished owing to the rapid development of the computer industry. Many researchers have carried out remarkable work on bubble dynamics with various methods (Guerri et al., 1981; Ji et al., 2014; Li et al., 2015; Zhang et al., 2015c). Among them, the Boundary Element Method (BEM) presents the high popularity for its simplicity by reducing one spatial dimension (Cheng and Cheng, 2005). Besides, conditions on the infinite boundary are satisfied automatically, which results in the BEM being outstanding while dealing with the infinite domain problems. For decades, many researches have been carried out to study the bubble dynamics through BEM (Blake and Gibson, 1987; Li et al., 2016; Wang and Manmi, 2014; Wang et al., 1996).

Suppose that N is the degrees of freedom (DOFs), the conventional BEM will produce an $N \times N$ asymmetrical and dense coefficient matrix for the discretized linear equations. For solving the equations, it will require the operations of $O\left(N^3\right)$ by direct solvers (e.g. the Gaussian Elimination), or $O\left(N^2\right)$ by the iterative solvers (e.g. the GMRES) (Saad and Schultz, 1986), which makes the computation rather expensive for large scale problems, let alone the simulation of dynamic problems with a couple of hundreds of time steps. These features are the drawbacks of BEM for analyzing large-scale problems, e.g., the air-gun array bubbles dynamics simulation. Numerous fast algorithms to accelerate the conventional BEM have been developed, including the Fast Multipole Method (FMM), the Fast Fourier Transform (FFT) based methods as well as the hierarchical methods such as the binary-tree or oct-tree structure methods. In our work, the FMM is chosen to achieve our goal.

The FMM, first demonstrated by Rokhlin and Greengard, is composed by the multipole and local expansions, which can be used to translate the evaluation of far field potential up/down through the tree structure (Greengard, 1988; Greengard and Rokhlin, 1987; Rokhlin, 1985). When FMM is applied to BEM, it is called the fast multipole boundary element method (FMBEM), and can enhance the computational efficiency dramatically. Compared with conventional BEM, it reduces the computational complexity from $O\left(N^3\right)$ or $O\left(N^2\right)$ to $O\left(N\right)$. Based on the efficient fast algorithms, BEM models with more than 1 million DOFs can be solved on a personal computer successfully (Nishimura, 2002). However, for large scale problems with complex geometries or moving-boundaries in time domain, the solution procedure still faces many challenges and difficulties. Among them, the evolution of multiple bubbles in the fluid is a typical example.

In early versions of Rokhlin and Greengard's FMM (Greengard and Rokhlin, 1987; Rokhlin, 1985), the "adjacent list" and the "interaction list" were introduced to get the contributions from far away cells to some certain cell in the same level of the tree structure. In the adaptive versions developed later (Carrier et al., 1988; Cheng et al., 1999; Shen and Liu, 2007), the "adjacent list" was optimized by other two lists to reduce the computing times in the direct calculation stage. Bapat and Liu (2010) further refined the "interaction list", and named their algorithm as "a new adaptive FMM" to distinguish it from the previous adaptive versions, which reduced the computations in the second most computing expensive part (apart from the direct calculation), i.e. the Multipole to Local translation (M2L). In this paper, our algorithm is based on Liu's refined version, and the exponential expansion is also adopted (Bapat and Liu, 2010; Greengard and Rokhlin, 1997; Shen and Liu, 2007). The details of FMM can be been in the work of Cheng et al. (1999), Liu (2009) and Yoshida (2001).

The Fast Fourier Transform on Multipoles (FFTM) is an FFT-based method first presented by Ong et al. (2003), which reduced the computing cost to $O\left(N^a\right)$, with a ranging from 1.0 to 1.3. The M2L translations are conducted by evaluating series of discrete convolutions with FFT algorithms. Since the empty cells have to be calculated, efficiency decreases when the sources are sparsely distributed. Bui et al. (2006) and TU (2005) introduced a new version of FFTM with clustering in solving moving boundary problems, and applied in the multiple bubbles dynamics simulations. The bubbles were firstly divided into groups according to their initial positions, with the intergroup evaluation conducted by M2L translations directly. Within each group, original FFTM is adopted. The FFTM-clustering method offers a satisfactory efficiency when bubbles are distributed sparsely. However, studies on multiple bubbles have not addressed bubbles with 3-D distribution and their dynamics under the gravity effect.

To our best knowledge, literatures focusing on air-gun array bubbles simulation with fast algorithm are limited. During the oscillation of the bubbles, the boundaries get closer, and the convergence of the iterative solver (e.g. the GMRES) used in FMBEM becomes slower, which may lead to the interruption of the simulation (Liu and Shen, 2007). A better conditioning of the equations may improve this situation, which can be achieved by combining the regular boundary integral equation (BIE) with the gradient BIE to form the dual BIE (Chen and Hong, 1999; Liu, 2009; Liu and Shen, 2007). In this paper, a linear dual BIE is used, see Section 4.1.

The paper is organized as follows. We start by reviewing briefly the formulations of the bubble dynamics and two kinds of BIEs as well as some basic FMBEM equations. Then, parameters of the dual FMBEM are chosen to optimize the algorithm. Numerical validations are then presented to verify the accuracy. Comparisons with the conventional BEM are conducted to show the efficiency of the dual FMBEM. Furthermore and most importantly, several cases of different distributions of the air-gun array are simulated by the dual FMBEM to study the complex evolution of multiple bubbles. Finally, a summary and concluding remarks of the work are presented.

2. The formulations of bubble dynamics

2.1. Physical equations of the bubble dynamics

Consider high pressure gas bubbles in the fluid domain Ω, which are initiated after triggering the air-guns. The origin of the