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Water wave interaction with two symmetric inclined permeable plates

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A B S T R A C T

We consider the reflection and the transmission of the surface water waves by two inclined identical permeable plates that are symmetric with respect to a line which is perpendicular to the horizontal line connecting the middle points of the plates. The problem is reduced to finding the solutions of two hypersingular integral equations of the second kind for the discontinuity in the velocity potential across one of the plates. The unknowns of the integral equations are expanded in terms of a finite series of Chebyshev polynomials multiplied by an appropriate weight function and then solved by collocating at suitable discrete points. Using these solutions, the numerical estimates for the reflection and the transmission coefficients, amplitudes of the hydrodynamic forces and moments and the wave dissipation coefficient are computed. Numerical results for these quantities are presented graphically in a number of figures. These results are compared with the published results for a single vertical as well as horizontal permeable plates and a submerged horizontal impermeable plate.

1. Introduction

Breakwaters are constructed to save harbours, beaches, shoreline, channel entrance and other coastal structures from the hazards caused by the rough sea. They are generally designed in such a way that maximum part of the wave energy can be reflected back. Sometimes, the reflected wave height is required to be small so that the possible occurrence of the wave resonance inside the sheltered area can be minimised. Porous structures are effectively used to protect shorelines as they reduce both the transmitted and the reflected wave heights due to their ability of reflecting, absorbing and dissipating wave energy.

As a thin vertical barrier can be regarded as an effective model of breakwater, study of its effect on the scattering of surface water wave is important. Ursell (1947) obtained the explicit solution for the scattering problem involving a surface piercing barrier. This problem had been re-examined experimentally as well as theoretically by Wiegel (1960). Evans (1970) studied the water wave scattering by a submerged thin vertical plate by converting the problem into a Riemann–Hilbert problem. Porter (1972) employed a complex variable technique together with an integral equation formulation to tackle the problem of wave transmission through a small aperture.

Based on the linear theory of water waves, a large number of theoretical research papers have been published on water wave scattering by thin porous structures vertically placed inside the sea. Tuck (1975) analysed the wave transmission through slots in a thin plate vertically placed in deep water. The relevant problem was numerically solved by Macaskill (1979). More studies on this topic have been carried out by means of the eigenfunction expansion method (cf. Losada et al., 1992; Abul-Azm, 1993), boundary element methods (cf. Liu and Abbaspour, 1982; Hsu and Wu, 1998), hypersingular integral equation technique (cf. Gayen and Mondal, 2014). Liu et al. (2007) examined the hydrodynamic performance of a new perforated-wall breakwater consisting of a perforated front wall, a solid back wall and a submerged horizontal porous plate installed between them using the matched eigenfunction method. Evans and Peter (2011) have employed the Wiener–Hopf method and the residue calculus technique to compute the reflection coefficient for semi-infinite and finite submerged horizontal porous plates respectively. Water wave interaction with horizontal permeable plate can also be found in the research articles of Liu and Li (2011), An and Faltinsen (2012), Cho and Kim (2013) and the references therein.

Sometimes it is necessary to use more than one barrier placed at suitable separation in order to reduce transmitted wave height in a considerable amount. The problems of water wave scattering by two thin and impermeable vertical barriers of different configurations were studied by Levine and Rodemich (1958) and Jarvis (1971) using the Schwarz–Christoffel transformation of complex-variable theory, by Newman (1974) employing matched asymptotic expansion method, by Das et al. (1997) using one-term Galerkin approximation technique, by Neelamani and Vedagiri (2002) with the help of some experimental

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investigations and by De et al. (2009, 2010), using the Abel integral equation approach. In the field of porous structures, the problems on the scattering of water waves by double or multiple porous structures are somewhat scarcely available in the literature. Twu and Lin (1991) investigated the performance of a highly effective wave absorber containing multiple porous plates with various porous-effect parameters. The research articles involving double permeable structures can be found in Losada et al. (1992), Isaacson et al. (1999), Koraim et al. (2011). Recently Karmakar and Soares (2014) studied gravity wave interaction with the multiple vertically moored bottom-standing flexible porous breakwaters with different edge conditions using the direct method and the wide-spacing approximation technique.

John (1948) considered a surface piercing plate making an angle $\pi/2n$ with the horizontal; n being an integer, using the reduction method to obtain the explicit solution. However the method was unwieldy except for $n=1$. Parsons and Martin (1992) developed a theory based on the numerical solution of a first kind hypersingular integral equation to deal with water wave scattering by a thin plate of an arbitrary inclination present in deep water. Their work was extended to the case of water of uniform finite depth by Midya et al. (2001). Gayen and Mandal (2003) investigated scattering of small amplitude waves past two symmetric inclined plates submerged in finite depth water. The three-dimensional problem of scattering of water waves by a thin disc submerged in deep water was investigated by Ziebell and Farina (2012). Yeung and Nokob (2013) solve (both analytically and numerically) the problem involving the wave scattering by a finite-draft surface piercing cylindrical shell at high and low frequency using hypersingular integral equation formulation.

Examples of water wave scattering by a permeable inclined plate can be found in Cho and Kim (2008) who studied the reflection of surface water waves by an inclined perforated plate placed in front of a vertical wall. Rao et al. (2009) experimentally studied the water wave transmission by a submerged inclined plate breakwater in regular waves and obtained that the wave energy was effectively reduced when the angle of inclination of the plate is 60°. Other significant experimental results regarding water wave interaction with inclined thin permeable plates are reported in Acanal et al. (2013), Yagci et al. (2014), Shih et al. (2015).

In our present work, we have considered two identical inclined plates symmetrically placed about the vertical line passing through the center of the horizontal line connecting the middle points of the plates. The plates are infinitely long in one horizontal direction taken as z-direction (cf. Fig. 1). As we consider the two dimensional motion in xy-plane, a cross section of each of the plates in this plane may be regarded as a thin plate. The plates may be supported by elastic cables or steel rods at the corners of the plate (cf. McIver, 1985; Acanal et al., 2013). As our purpose is to determine the reflection and the transmission coefficients at a large distance away from the plates,

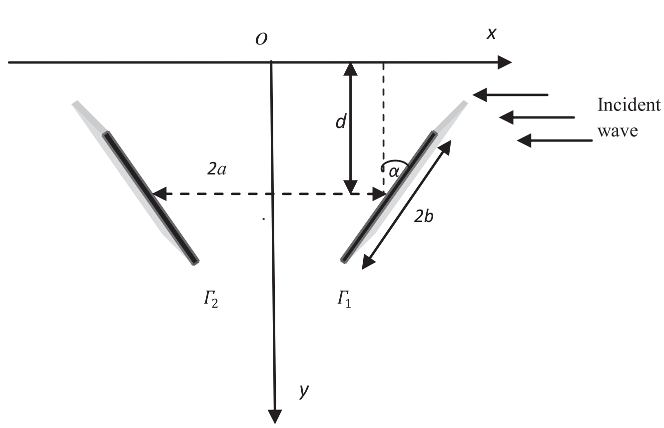

Fig. 1. Geometry of the inclined porous plates.

it is assumed that the presence of the supports does not affect the reflected or the transmitted wave fields. Exploiting the geometrical symmetry of the plates about y-axis, the problem is reduced to two boundary value problems in the symmetric and the antisymmetric components of the potential functions. Employing Green's integral theorem and the boundary conditions on the porous plates, both the problems are reduced to two integral equations with hypersingular kernels; the unknowns of these equations being the differences in potential functions across a plate. Using the behaviour of the potential functions at the tip of the porous plates, the unknown functions are expanded in terms of a finite series and then reduced to two systems of linear equations by collocating at finite number of points using zeros of the Chebyshev polynomial of the first kind. The linear equations are solved to determine the numerical solutions of the discretised values of the potential differences which are further used to evaluate the numerical estimations of various physical quantities viz., the reflection and the transmission coefficients and the hydrodynamic forces and moments acting on the plates. A mathematical expression for the energy identity relevant to the inclined porous plates is derived. The numerical results for the reflection and the transmission coefficients are validated against this identity. The correctness of the numerical results is also verified by comparing them with the well-known results for a single vertical permeable plate, single horizontal permeable and impermeable plates. Furthermore, a brief outline of the process of employing the present integral equation approach to study a mooring system is discussed.

2. Mathematical formulation

We choose a Cartesian co-ordinate system with the mean free surface along $y=0$. The fluid of infinite depth occupies the region $0 < y < \infty$, $-\infty < x, z < \infty$. Two permeable plates Γ_1 and Γ_2 with similar properties, each of length $2b$ are placed symmetrically with respect to the y-axis as shown in Fig. 1. As mentioned in the Introduction, we consider a cross-section of the plates in the xy-plane and assume the plate to be thin. The distance between the middle points of the plates is $2a$. The depth of submergence of the middle points of the plates below the x-axis is taken as d. The plate Γ_1 is inclined at an angle α with the upward vertical and further, a, b and α must satisfy the inequality $a > b\sin\alpha$ and $d > b\cos\alpha$. Regarding the fluid flow, we assume that it is irrotational and can be described using linear inviscid theory.

It is assumed that a wave of frequency $\sigma/2\pi$ is incident from $x=+\infty$. Then it follows from the linearised theory of water waves that, there exists a velocity potential $\Phi(x, y, \tau)$ which may be expressed as the real part of a complex quantity

$$\Phi(x, y, \tau) = \mathrm{Re}\left\{\frac{g^2}{\sigma^3}\phi(x, y)e^{-i\sigma\tau}\right\} \tag{2.1}$$

where g is the gravitational constant and τ denotes the time. The factor g^2/σ^3 is multiplied with $\phi(x, y)$ to make it non-dimensional.

Then $\phi(x, y)$ must satisfy the governing Laplace equation in the fluid region

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \text{ on } -\infty < x < \infty,\ 0 < y < \infty. \tag{2.2}$$

The linearised boundary condition on the free surface reads

$$K\phi + \phi_y = 0 \text{ on } y = 0, \tag{2.3}$$

where $K = \sigma^2/g$ is the wavenumber. The no flow condition at the rigid bottom is given by,

$$\nabla\phi \to 0 \text{ as } y \to \infty. \tag{2.4}$$

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