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Investigation on the probabilistic distribution of mooring line tension for fatigue damage assessment

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ABSTRACT

This paper is to investigate the applicable probability density functions (PDFs) for the short- and long-term mooring line tension amplitudes to be used for accurately assessing the mooring line fatigue damage with the spectral method. Statistical characteristics of the mooring line tensions were investigated, in which kurtosis and skewness were computed and compared to validate the non-normality of the line tension response. Several PDFs, including Rayleigh, GEV, and Gamma distributions, are used to express the short-term statistical distribution of the mooring line tension amplitudes. Results indicate that the Gamma distribution can give much better fitting than the Rayleigh and GEV distributions. In order to identify the long-term distribution of the tension amplitudes, the Gamma distribution is modified by bringing in a location parameter. Compared with the generally used Weibull distribution, the modified Gamma distribution can characterize the long-term PDF of the tension amplitudes much better. Moreover, the modified Gamma distribution can give much more accurate assessment of the mooring line fatigue damage than the Weibull distribution.

1. Introduction

The spectral method and the time domain analysis method (Ariduru, 2004) are the most important techniques to assess the fatigue damage of offshore structures, and the assessment accuracy is always benchmarked against the result from the time-domain analysis method along with its associated rainflow counting technique (Matsuishi and Endo, 1968; Rychlik, 1987). However, the calculation procedure of the time domain analysis is pretty complicated and time consuming. Compared to the time domain analysis method, the spectral method has significant advantages, e.g., it needs less computation time and storage space.

The structural response amplitude/range distribution is necessary for the spectral fatigue damage assessment method. Linear structural responses due to waves can usually be assumed to be Gaussian and narrow-banded in marine technology and offshore engineering (Gao and Moan, 2008). As a result, the stress amplitudes in each short-term stationary sea state can be described with the Rayleigh distribution, and the long-term statistics by the Weibull distribution. However, for the mooring line system, the line tension is in principle non-Gaussian and its spectrum is typically wide-banded due to the nonlinearities of the mooring line characteristics and the second-order responses (Gao and Moan, 2007). In consequence, the traditionally used Rayleigh and Weibull functions are not suitable, and both of the distributions would cause obvious errors when estimating the mooring line fatigue damage with the spectral method (Low, 2010, 2011).

In order to accurately estimate the fatigue damage induced by the non-Gaussian stress process, two main approaches have been studied. One is to find a bandwidth correction factor to modify the narrow-band approximation, and many researchers, including Wirsching and Light (1980), Dirlik (1985), Rice (1944), Kim et al. (2007), Braccesi et al. (2005) etc., have done much work on this topic. Benasciutti and Tovo (2005) discussed and compared these methods with the time domain analysis method, and concluded that the method proposed by Dirlik (1985) is the best one. However, Dirlik's method has some theoretical limitations (Dirlik, 1985; Benasciutti and Tovo, 2005).

The other approach is to focus on the PDF of the response amplitude. Mooring lines are typically subjected to bimodal loads, consisting of a wave frequency (WF) component due to the first order wave forces and a low frequency (LF) component induced by the second-order wave forces (Gao and Moan, 2007). When both of the WF and LF processes are Gaussian, Jiao and Moan (1990) approximated the combined fatigue damage as a sum of the WF fatigue damage and the damage due to a process consisting of the LF process and the envelope of the WF process. They developed the theoretical distribution models for the WF and LF mooring line

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tension, respectively. Meanwhile, the mooring line fatigue damage was estimated based on these distribution formulae, and the results showed good. Compared to the bimodal process of mooring line tensions, there is one more component in tensions of marine risers, which is a tri-modal process: a LF response due to the riser pre-tension, a mid-range excitation frequency (MF) response from ambient waves and a high frequency (HF) excitation due to vortex induced vibration (VIV). Gao and Moan (2008) defined the ideal tri-modal process as the sum of three narrow-band random processes with well separated central frequencies, and studied the riser fatigue damage caused by the ideal tri-modal process. The results indicated that when the bandwidth is greater than 0.85, the tri-modal formulation becomes inaccurate and will significantly overestimate the fatigue damage. Park and Song (2015) proposed ideal tri-modal spectrum formulations to compare various fatigue damage models developed to predict fatigue damage for wide band spectrum through theoretical or empirical approaches. They concluded that the Benasciutti-Tovo (Benasciutti and Tovo, 2005) and JB (Park et al., 2014) models represented the most appropriate fatigue damage for ideal tri-modal spectrum with well separated central frequencies. All the previous works have made a great contribution to the mooring line fatigue damage assessment. However, it is assumed that the bimodal and tri-modal processes can be well separated to Gaussian and narrow-banded components with significant central frequencies. For the mooring line system subjected to environmental loads, the line tension variation is generally a non-Gaussian process and its spectrum is typically wide-banded due to the nonlinearities of the mooring characteristics and the drag force acting on the line.

If a short-term process, such as the linear response of the floater under a stationary sea state, is assumed to be Gaussian, then its long-term distribution of response amplitude (also for its response range) can be expressed with the Weibull distribution (Dong et al., 2011). To the author's best knowledge, there is a very few literatures about the long-term distribution for the non-Gaussian progress of the offshore structure stress or mooring line tension amplitudes. An example is the research by Dong et al. (2011), in which the long-term statistical distribution of the hot-spot stress amplitudes of a fixed wind turbine, which is loaded by wind and wave simultaneously, was numerically simulated. The results indicated that the Weibull function can be utilized to fit the long-term statistical distribution of the turbine stress amplitudes for the case of wave loads only. But for the cases of wind loads only and the combination of wave and wind loads, the generalized gamma distribution becomes better than the Weibull distribution.

In order to overcome the drawbacks of the commonly used Rayleigh and Weibull distributions due to the non-normality of the mooring line tension response, this study will deeply investigate the short- and long-term statistical characteristics of the mooring line tension amplitudes to further seek more appropriate distribution function for accurate fatigue assessment with the spectral method. The Gamma distribution is introduced to describe the short-term distribution of the mooring line tension amplitudes, and further modified to characterize the long-term distribution. Based on the Gamma and modified Gamma distributions, the mooring line fatigue damage is estimated with the spectral method and compared with the results from the commonly used distributions, in which the fatigue damage assessed with the time domain analysis method is considered as the benchmark. To this end, the remainder of this paper is organized as follows. Section 2 describes several distribution patterns, including the Gamma and the modified Gamma distribution. The commonly used fatigue assessment methods are presented in Section 3. In Section 4, the numerical simulations are executed to investigate the non-Gaussian nature of the mooring line tension response, and to validate the Gamma distribution and the modified one for calculating the mooring line fatigue damage with the spectral method. Finally, the main conclusions drawn from this study are listed in Section 5.

2. Distribution for structural response

2.1. Gaussian distribution

Gaussian distribution is also called Normal distribution which has two characteristic parameters including the mean value and the standard deviation. And the PDF of the Gaussian distribution is shown in Eq. (1).

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{1}$$

in which, x is the variable; μ is the mean value of x; and σ is the standard deviation of the variable.

The Gaussian distribution is generally used in the fields of mathematics, physics and engineering. In the ocean engineering field, the wave elevation and the linear structural responses can be well fitted by the Gaussian distribution.

2.2. Rayleigh distribution

The linear responses of offshore structures and ships subjected to wave loads are always considered as Gaussian processes. As a result, the short-term response amplitudes follow the Rayleigh distribution and the PDF can be expressed as Eq. (2),

$$f(x) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right), \quad x \ge 0 \tag{2}$$

in which, x is the amplitude of the variable, and σ is the standard deviation of the variable.

2.3. General extreme value distribution

The general extreme value distribution (GEV), which is comprised of Gumbel, Frechet and Weibull distribution, is commonly used to describe the distribution of the extreme climate elements (Kim et al., 2012). And the PDF of the GEV distribution is,

$$f(x) = \frac{1}{\sigma}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{(-1/\xi)-1}\exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \tag{3}$$

in which, μ is the mean of the variable and it is named as the location parameter, σ is the standard deviation which is called as scale parameter and ξ is the shape parameter.

When the line tension range (two times of the amplitude) is considered as the extreme value of a certain short duration, the GEV can be used to fit the distribution of the line tension amplitudes for the purpose of the comparison.

2.4. Weibull distribution

The two-parameter Weibull distribution, whose PDF is shown in Eq. (4), has been extensively used over the past decades for fitting data in reliability, engineering and biological studies (Castellares and Lemonte, 2014).

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\exp\left[-\left(\frac{x}{\lambda}\right)^{k}\right] \quad x \ge 0 \tag{4}$$

in which, x is the variable; k and λ are the shape and scale parameters respectively, and they have the relation with the mean μ and standard deviation σ of the statistics as shown in Eq. (5):

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