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Extreme value estimation using the likelihood-weighted method

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ABSTRACT

This paper proposes a practical approach to extreme value estimation for small samples of observations with truncated values, or high measurement uncertainty, facilitating reasonable estimation of epistemic uncertainty. The approach, called the likelihood-weighted method (LWM), involves Bayesian inference incorporating group likelihood for the generalised Pareto or generalised extreme value distributions and near-uniform prior distributions for parameters. Group likelihood (as opposed to standard likelihood) provides a straightforward mechanism to incorporate measurement error in inference, and adopting flat priors simplifies computation. The method's statistical and computational efficiency are validated by numerical experiment for small samples of size at most 10. Ocean wave applications reveal shortcomings of competitor methods, and advantages of estimating epistemic uncertainty within a Bayesian framework in particular.

1. Introduction

Extreme value estimation characterizes the tail of a probability density distribution, and often requires extrapolation beyond what has been observed (Coles et al., 2001). Extrapolation is motivated by extreme value theory for the asymptotic distribution of large values from any max-stable distribution. A basic assumption in fitting an extreme value model to a sample is that observations are independently and identically distributed. This assumption usually holds for the rarest and severest of ocean wave events (e.g. the storm peaks over threshold of significant wave heights in a tropical cyclone at a location). The trade-off between sample size and adequate tail fit, and the fact that measurement errors on most extreme observations tend to be large, render the analysis problematic.

The increasing availability of high quality measurements and hindcasts means that the metocean engineer is often blessed with huge samples for estimation of return values for design purposes. Extreme value modelling is then a large-scale computational task, within which the effects of non-stationarity and spatial dependence can be estimated (Jonathan and Ewans, 2013). However, there are many other applications where large samples of high quality data are still not available. The metocean engineer is then required to provide design values from small samples of typically poor quality. For such analysis, uncertainties in extreme value parameters and return value estimates are large and often difficult to estimate well. The effective number of influential observations in estimating extreme events with very low probability, such as at the ten thousand year return period level, may be small even in samples corresponding to a hundred years of observations. The goal of this paper is to explore a method for extreme value estimation useful for small samples (of size at most 10) of poor quality data, which provides realistic estimation of epistemic model uncertainty. The approach, called the likelihood-weighted method (LWM), involves Bayesian inference for the group generalised Pareto (or generalised extreme value) likelihood and uniform prior distributions for parameters. Group likelihood provides a straightforward mechanism to incorporate measurement error; adopting flat priors simplifies computation.

Statistical models exhibit two types of uncertainty (Bitner-Gregersen and Skjong, 2009). Aleatory uncertainty represents the inherent randomness of nature and physics; it is intrinsic and cannot be reduced. Epistemic uncertainty represents our limited knowledge, and can be reduced (e.g.) by increasing sample size or reducing sample measurement error. Realistic estimation of epistemic uncertainty is critical to reliable extreme value modelling. We will demonstrate that estimation methods such as maximum likelihood provide poor estimates of epistemic uncertainty from small samples of poor quality.

The organisation of the article is as follows. In Section 2, we review methods in extreme value analysis with emphasis on uncertainty quantification from poor data. A description of LWM, our new estimation method, is given in Section 3. In Section 4, LWM's

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statistical and numerical efficiency is validated through numerical experiments. An application to observed extreme wave height data is considered for further discussion in Section 5, followed by conclusion in Section 6.

2. Extreme value estimation for small samples measured with error

2.1. Extreme value theory

The central limit theorem provides an asymptotic distributional form (the Gaussian distribution) for the mean $A_n \left(= (1/n) \sum_{j=1}^{n} X_j \right)$ of n independent observations of identically-distributed random variables $X_1, X_2, \ldots, X_n$, regardless of the underlying distribution. Analogously, extreme value theory provides an asymptotic distributional form for independent observations from any of a large class of so-called max-stable distributions (Kotz and Nadarajah, 2000). The limiting forms for extreme values of block maxima M_n $(= \max(X_1, X_2, \ldots, X_n))$ were given by Jenkinson (1955), and were later rationalised into one generalised extreme value (GEV) distributional form. Pickands (1975) and Balkema and De Haan (1974) derived the generalised Pareto (GP) distribution for peaks over threshold (POT) by considering the logarithms of the GEV.

GEV and GP are three-parameter distributions, with parameters shape ξ, scale σ and location μ (for GEV) or extreme value threshold ψ (for GP). Cumulative distribution functions (cdfs, F_{GEV} and F_{GP} respectively) for these distributions are given in Eqs. (1) and (2). Other distributional forms are used for extreme value estimation, including the Weibull and log-normal distributions e.g. Ochi (2005), Muir and El-Shaarawi (1986). Here we focus on GEV and GP, given their natural asymptotic motivation and wide application:

$$\Pr(M_n \le x) \overset{\text{large } n}{\approx} F_{GEV}(x) = \exp\left(-\left(1 + \frac{\xi}{\sigma}(x-\mu)\right)^{-1/\xi} \right) \quad \text{for } \xi \neq 0 \qquad = \exp\left(-\exp\left(-\frac{1}{\sigma}(x-\mu) \right) \right) \quad \text{otherwise, and} \tag{1}$$

$$\Pr(X \le x | X > \psi) \overset{\text{large } \psi}{\approx} F_{GP}(x) = 1 - \left(1 + \frac{\xi}{\sigma}(x-\psi)\right)^{-1/\xi} \quad \text{for } \xi \neq 0 \qquad = 1 - \exp\left(-\frac{1}{\sigma}(x-\psi) \right) \quad \text{otherwise.} \tag{2}$$

These distributional forms are correct asymptotically for block maxima and peaks over threshold, but only approximately for finite samples. Increasing sample size for fitting is desirable to reduce estimated parameter bias and uncertainty, but often is achieved at the expense of quality of fit of an extreme value distribution to the largest values in the sample (e.g. by reducing block size for GEV, or reducing extreme value threshold for GP). We do not address this trade-off directly in this work; rather, we assume that the sample is drawn from the extreme value distribution to be estimated, and concentrate on estimating parameters and uncertainty.

2.2. Parameter estimation

There are many possible approaches for parameter estimation in extreme value analysis. Popular schemes include maximum likelihood (ML), the method of moments, probability weighted moments (PWM), L-moments and Bayesian inference (Muir and El-Shaarawi, 1986). Graphical methods have also been proposed, but these are not recommended for quantitative work. Other empirically-derived estimation methods such as Goda's method (Wada and Waseda) lack generality. Methods based on moments or likelihoods are most common in the literature (Palutikof et al., 1999).

For small samples, moment-based methods such as PWM and L-moments, are considered better than ML (in terms of bias and mean square error) for point estimation of parameters (Hosking et al., 1985). Here our interest is not in deriving point estimates, since large epistemic uncertainty is obviously unavoidable, and quantification of the epistemic uncertainty of much greater importance. For both ML and PWM, confidence intervals can be estimated by the so-called delta method, or the profile likelihood method; both are motivated by consideration of asymptotic behaviour, and strictly valid for large samples. Smith and Naylor (1987) considers extreme value estimation for a three-parameter Weibull distribution using maximum likelihood for sample size of over 40, and discusses the resulting unusual likelihood shape. In some applications, even a sample size as small as 20 is difficult to gather. This is the motivation for the current work: we focus on extreme value estimation from sample sizes of at most 10.

Resampling methods such as bootstrapping are also used for uncertainty quantification. The simplest resampling scheme draws random re-samples with replacement from the original sample (Efron, 1979, is easy to implement and widely used. Uncertainty quantification from resampling is rather ad hoc in nature, certainly compared with Bayesian inference. We will illustrate the shortcomings of a simple bootstrap method for small samples in Section 4.

2.3. Bayesian inference

Bayesian methods exploit both the sample likelihood and prior distributions for parameters in inference. The favourable performance of Bayesian inference in extreme value estimation from small samples has been discussed (Coles and Powell, 1996). One advantage of the Bayesian approach is the flexibility offered to estimate unusually shaped likelihood surfaces (Smith and Naylor, 1987).

The basic equations of Bayesian inference are described below. The sample likelihood $L(\theta; D)$ of parameter(s) θ for sample $D = \{x_i\}_{i=1}^{n}$ is interpreted as the probability of the sample given parameters

$$f(D|\theta) = L(\theta; D) = \prod_{i=1}^{n} f(x_i|\theta). \tag{3}$$

The probability of the sample is then

$$f(D) = \int_{\theta} f(D|\theta) dF(\theta), \tag{4}$$

where we can interpret $dF(\theta)$ as $f(\theta)d\theta$ for continuous prior density $f(\theta)$ for θ. We estimate the posterior distribution of θ using Bayes theorem

$$f(\theta|D) = \frac{f(D|\theta) f(\theta)}{f(D)}. \tag{5}$$

The posterior $f(\theta|D)$ can be used, amongst other things, to estimate credible intervals for parameters. The posterior predictive distribution $g(x|D)$ of any function $g(x|\theta)$ is then the expected value of that function under the posterior distribution $f(\theta|D)$ for θ. The posterior predictive distribution therefore captures both epistemic and aleatory uncertainty

$$g(x|D) = E_{\theta|D}\Big(g(x|\theta) \Big) = \int g(x|\theta) f(\theta|D) d\theta. \tag{6}$$