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On the use of Laplace's equation for pressure and a mesh-free method for 3D simulation of nonlinear sloshing in tanks

B. Boroomand [a,*], S. Bazazzadeh [b,c], S.M. Zandi [d]

[a] Department of Civil Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran
[b] Department of Industrial Engineering, University of Padova, Via Venezia 1, 35131 Padova, Italy
[c] Center of Studies and Activities for Space "Giuseppe Colombo", CISAS, Via Venezia 15, 35131 Padova, Italy
[d] Department of Civil Engineering, University of Isfahan, Isfahan 81744-73441, Iran

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ABSTRACT

In this paper, it is first proven that instead of Poisson's equation one can use Laplace's equation for the pressure, which is much simpler to solve, in Lagrangian simulation of incompressible inviscid Newtonian fluid flow problems starting from a divergence-free initial acceleration condition. When Laplace's equation for the pressure is used in Newmark time integration scheme it guarantees mass conservation with $O(\Delta t^3)$ accuracy. Next in this paper a consistent 3D mesh-free method for the solution of free surface sloshing in tanks is presented. In this method a linear summation of exponential basis functions (EBFs) is assumed as an approximation to the solution. The coefficients of the series are determined by a collocation technique used on a set of boundary nodes. These coefficients and the surface boundary nodes are updated through a time marching algorithm. Linear/non-linear 3D sloshing problems are solved in both rectangular and cylindrical basins. It is shown that the method may be used as an effective tool for 3D simulation of tanks with various shapes without the need for a huge number of domain/boundary elements for the discretization.

1. Introduction

Free surface fluid flows with moving boundaries are of concern in many engineering problems as for instance sloshing tanks, dam-reservoir interaction, tuned liquid dampers, ballast tanks of ships etc. (see for instance Mitra et al., 2012). Among the challenging issues in the field of fluid dynamics are those in which the boundaries of the fluid vary during the numerical solution.

Prediction of the free surface of water in tanks, e.g. ballast tanks of ships, becomes prominent especially when the sloshing frequency is near the first natural frequency of the fluid-tank system (see also the studies by Zhang et al. (2015) for "second-order resonance" happening due to the interaction among natural frequencies and excitation frequencies). It is also crucial to precisely estimate the forces on the walls of the tanks in order to prevent damage. Efficient simulation tools are then needed during the design of such structures.

In the simulation processes, two well-known descriptions are generally used for the fluid flows, i.e. Eulerian description and Lagrangian description. In a Eulerian description (see Zienkiewicz and Taylor (2000) for instance) a fixed computational grid is considered and the fluid is permitted to flow through the grid. In a Lagrangian description, however, the computational grid moves with the fluid domain and its boundaries. This approach has recently received considerable attention especially in the solution of free surface flow problems (Ramaswamy and Kawahara, 1986, 1987a; Radovitzky and Oritz, 1998; Idelsohn et al., 2003; Shingareva and Celaya, 2007).

In the literature, one can find an intermediate approach, known as arbitrary Lagrangian–Eulerian (ALE), which allows the grid points to move independently of the fluid. In this line, the studies on free surface fluid problems may be traced in the research works by Ramaswamy and Kawahara (1987b), Lo and Young (2004), Durate et al. (2004) and Nithiarasu (2005). The readers may also refer to the studies by Zhang (2015) for the recent studies on using Lagarngian description and on an improved version of the so called "semi-Lagrangian" formulation.

In the realm of analytical solutions and experimental results for sloshing tanks, the studies by Faltinsen (1978) and Faltinsen et al. (2005) on 2D/3D linear and nonlinear sloshing problems should be mentioned here. More experimental/numerical investigations may be found in the works by Akyildiz and Unal (2005, 2006). In the absence of analytical solutions in many problems, the use of

* Corresponding author.
E-mail address: boromand@cc.iut.ac.ir (B. Boroomand).

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numerical solution tools seems to be inevitable. In the realm of numerical simulation of linear/nonlinear sloshing in tanks, one may trace the studies by Frandsen (2004), Chen (2005), Chen and Nokes (2005), Sames et al. (2002), Chen et al. (2007), Liu and Lin (2008), Wu and Chen (2009), Chen and Wu (2011), Wu and Chang (2011), and Zandi et al. (2012a).

In the aforementioned studies on sloshing tanks, a trend towards eliminating mesh construction in the computational process is observed. For instance, in the study by Chen et al. (2007) on tanks containing inviscid fluids, the boundary element method (BEM) was employed to reduce the labor of generating a domain mesh of elements in a 3D analysis. However, it is well understood that the BEM needs surface meshes and thus it still needs mesh generation at the boundaries of the domain. In the same line, using meshfree methods (again for inviscid fluids), Wu and Chang (2011) employed radial basis functions (RBFs) to model the 3D fluid just by a set of domain points. A collocation approach was used at the domain points and the boundary points to satisfy the governing equations and the boundary conditions.

Most of the studies on sloshing of incompressible fluids either directly use the Navier-Stokes equations or follow a velocity potential approach. The direct use of the pressure has been reported in many studies using Lagrangian description (see for instance Idelsohn et al., 2001; Ma, 2005; Shobeyri and Afshar, 2010) in the form of Poisson's equation. In these studies the right hand side of the Poisson's equation in each time step is defined from the fictitious numerical compressibility found from the previous time steps. In a more simpler case, one may use Laplace's equation for the pressure as the only governing equation; however the use of such a Laplace's equation has been reported in a limited number of studies (see for instance Tang (1993), Mitra et al. (2008) or the latest one Zandi et al. (2012a) for 2D problems). Before defining such a Laplace's equation for the pressure, it must be shown that in a Lagrangain description, the divergence of the acceleration field vanishes. It appears to the authors of this paper that, especially for nonlinear fluid flow problems, some investigators cast doubt about the existence of a divergence-free acceleration field while the velocity field is divergence-free (although many numerical evidences clearly show that the results found from the Laplace's equation for the pressure match the analytical/experimental results). Therefore, as the first step in this paper, the validity of assuming divergence-free acceleration field for incompressible fluid flow problems is proven. To this end, first a Newmark algorithm for time marching solution is employed. It will be shown that the use of Laplace's equation for the pressure can provide mass conservation with $O(\Delta t^3)$ accuracy. Such a pressure-only governing equation paves the way for constructing an efficient algorithm for fluid-structure problems.

As the second step in this study, a boundary point meshless method is presented to simulate 3-D sloshing of incompressible inviscid fluids as the extension of the study performed by Zandi et al. (2012a) for 2D problems. No boundary-mesh of elements, as in the BEM for instance, and no domain points are needed for the solution. This advantage is effectively demonstrated in 3D problems which are the cases in this study. An approximate solution as a series of exponential basis functions (EBF) is assumed for the pressure of the fluid. The EBFs are chosen so that they satisfy the governing Laplace's equation in terms of the pressure. The Laplace's equation is then solved by defining boundary conditions of the fluid in terms of the wall accelerations and the free surface pressure. These boundary conditions are imposed by a collocation approach. The solution is performed through a Newmark time-marching algorithm in which the free surface of the fluid is updated incrementally. This is performed in an implicit form through defining an intermediate geometry. Rigid cubic and cylindrical basins are considered to present the numerical results.

As already mentioned, the basis of the method here in 3D cases, follows that previously presented by Zandi et al. (2012a) for 2D free surface flows. They tested the method in many linear and nonlinear problems especially when the oscillation is near resonance frequency. This method is categorized as Trefftz methods (see Li et al., 2007 and Movahedian et al., 2013). Other applications of this truly meshless method have been presented by Boroomand et al. (2010), Zandi et al. (2012b), Shahbazi et al. (2011a, 2011b, 2012), Abdollahi and Boroomand (2014), Azhari et al. (2013a, 2013b), Hashemi et al. (2013) and Movahedian and Boroomand (2014, 2015) which include plate problems, nonlocal elasticity, static, time harmonic elastic and direct/inverse heat conduction problems. The objective in this paper is to extend the method to non-linear 3D problems.

The layout of the paper is as follows. In Section 2, after describing the model used for incompressible inviscid fluid flow, the proof for the validity of Laplace's equation for the pressure is given. The solution procedure is explained in Section 3 which includes the definition of the EBFs for the Laplace equation in 3D and the method of using the complex discrete transformation technique for the imposition of the boundary conditions. A step-by-step summary of the procedure is given in the same section. In Section 4, some benchmark problems are solved and the numerical results are compared with those obtained from analytical methods and other numerical approaches to show the capability of the proposed procedure.

2. Governing Lagrangian equations

A 3D domain, Ω, occupied by an inviscid Newtonian fluid is considered. The domain boundaries $\partial\Omega$ include both fluid-tank interface Γ_S and the free surface of fluid Γ_F (Fig. 1). The governing equations of the motion are then momentum conservation, in the absence of viscosity, as

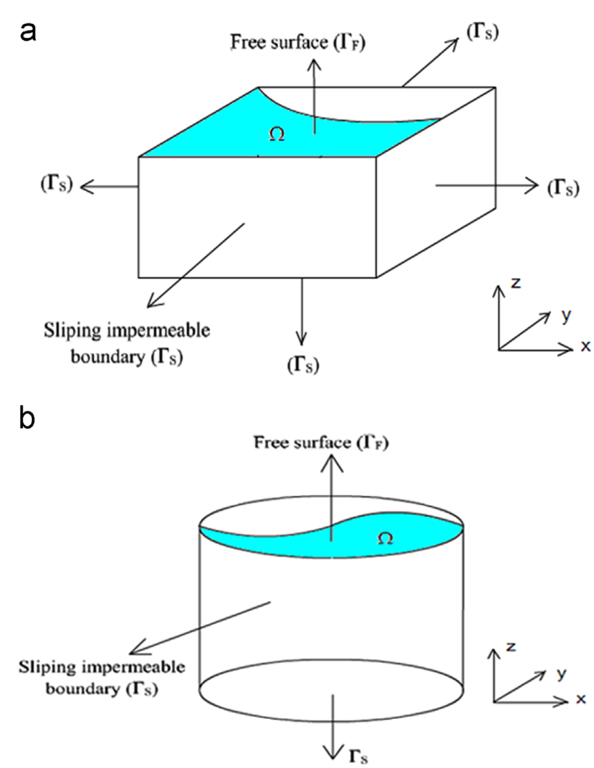

Fig. 1. Domain of the problem and its boundaries in two forms of tanks in this study.