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A note on the relative efficiency of methods for computing the transient free-surface Green function

Harry B. Bingham

Department of Mechanical Engineering, Technical University of Denmark, 2800 Lyngby, Denmark

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ABSTRACT

A number of papers have appeared recently on computing the time-domain, free-surface Green function. Two papers in particular, Chuang et al. (2007) and Li et al. (2015) considered the method developed by Clement (1998) who showed that this Green function is the solution to a fourth-order Ordinary Differential Equation (ODE). This ODE has been suggested as a means for speeding up the calculation of the Green function coefficients compared to the standard algorithms developed for example by Newman (1992). Clement solved the ODE using the classical fourth-order, four-step Runge–Kutta scheme (RK44) with a fixed time step size. The two papers mentioned above proposed alternative numerical methods which are claimed to be more efficient. In this note we consider the relative efficiency of these four methods on a representative test case, and conclude that the standard method is the most efficient. Of the ODE-based methods, the method of Chuang et al. (2007) is found to be slightly more efficient than the RK44 method, while the method of Li et al. (2015) is at least an order of magnitude less efficient. It is also pointed out that ODE methods have yet to be extended to include finite water depth.

1. Introduction

The transient free-surface Green function (see Wehausen and Laitone, 1960) is the basis for one class of time-domain panel method codes solving the Neumann–Kelvin linearized seakeeping problem. It is also useful for providing radiation boundary conditions in the far-field of more flexible methods which solve less restrictive versions of the seakeeping problem. By *seakeeping problem* we refer to the interaction between a ship traveling at steady forward speed U and the ocean waves, and we are concerned here with potential flow approximations. By *Neumann–Kelvin* linearization we refer to the linearized version of the problem where the steady disturbance created by the ship is assumed to be a small perturbation to the undisturbed streaming flow. See for example Newman (1977) and Ogilvie (1964) for more complete descriptions of this theory. A number of numerical solutions to this problem by means of boundary element methods based on the transient free-surface Green function have appeared over the years, see for example Beck and Liapis (1987), Liapis (1986), King et al. (1988), Korsmeyer (1988), Bingham et al. (1993), Korsmeyer and Bingham (1998), Clement (1998), and Kara (2011).

Efficient methods for accurately computing the transient free-surface Green function were initially developed by combining asymptotic series expansions with interpolating polynomials as described for example by Newman (1992). In 1997, Clement discovered that this Green function is the solution to a fourth-order Ordinary Differential Equation (ODE) Clement (1997, 1998). This ODE was envisioned as a means of avoiding the convolution integrals appearing in the time-domain formulation of the problem, and although this goal has not yet been realized, the ODE also provided an alternative means of computing the Green function. Clement adopted the classical explicit four-step, fourth-order Runge–Kutta scheme (RK44) to integrate the ODE and compute the Green function, as did several other subsequent authors including Duan and Dai (2001) and Liang et al. (2007). Ten years later, Chuang et al. (2007) developed a closed-form solution to the ODE based on Taylor series expansions, allowing the Green function to be computed by truncating the series at a finite number of terms. Although not explicitly stated, the implicit intent of this work was to provide a more efficient means of solving the ODE compared to the RK44 method, even though no demonstration of that result was provided. More recently, Li et al. (2015) showed that both the Green function and its gradient can be computed from the solution to the same ODE. They also applied the Precise Integration (PI) method of Liu et al. (2014) to solve the ODE, and showed that this was more stable and accurate than RK44 integration when the same time step size is used. They also imply that the method is more efficient than the Taylor expansion method of Chuang et al. (2007), though this was not demonstrated. Comparing two numerical methods at the same time-step size is however only

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relevant when the two methods require the same computational effort per time-step, which is not the case here. The real question of interest is: which method solves the ODE more efficiently? The answer to that question is found by measuring how much computational time is used by each method to achieve a given level of error. The current state of affairs is thus, that no conclusive demonstration has yet appeared to support the claim that the use of the Clement ODE can accelerate time-domain calculations of wave–body interaction. Neither is it clear which of the three currently recommended methods for solving the Clement ODE is the most efficient.

The goal of this note is to compare the relative efficiency of the four above mentioned methods for computing the time-domain free-surface Green function. This has been done by implementing all four methods in the same computer code, and comparing them on a representative test case where the exact solution is known. We have attempted to optimize each method so as to minimize the computational effort required to achieve a desired level of accuracy. Confirmation that the numerical implementation is reasonably optimal is found by recovering the theoretically expected relative scaling of the computational effort in each case. From this representative test case, we conclude that the standard method is the most efficient. Of the three methods for solving the Clement ODE, the Taylor expansion method is found to be slightly more efficient than the RK44 method with the PI method at least an order of magnitude less efficient than all of the other methods. We also point out that while the standard method supports finite water depth, this extension has yet to be made for the ODE representation. The promise of the Clement ODE for accelerating time-domain hydrodynamic calculations remains thus enticing but not yet realized.

2. The transient free-surface Green function and the generating ODE

In this section we review the definition of the Green function and the ODE that can be used to generate it. To establish some context, consider a generic ship represented by a collection of panels as shown in Fig. 1. Only the submerged portion of the geometry is shown and a Cartesian coordinate system is adopted with the z-axis vertical and the x–y plane at the still water level. Consider two points on this geometry $\boldsymbol{x} = (x, y, z)$ and $\boldsymbol{\xi} = (\xi, \eta, \zeta)$, where the physical variables have been non-dimensionalized by a length scale L (typically the ship length) and the gravitational constant g. In practice, these two points will be collocation points located on two of the panels describing the body surface. In deep water, the velocity potential at the field point $\boldsymbol{x}$ due to a unit-strength impulsive source at the source point $\boldsymbol{\xi}$ is given by (Wehausen and Laitone, 1960)

$$G(\boldsymbol{x};\, \boldsymbol{\xi},\, t) = \delta(t) G_0(\boldsymbol{x};\, \boldsymbol{\xi}) + H(t) \tilde{F}(\boldsymbol{x};\, \boldsymbol{\xi},\, t) \quad (1)$$

where $\delta(t)$ is the Dirac delta function and $H(t)$ the Heaviside step function. Here

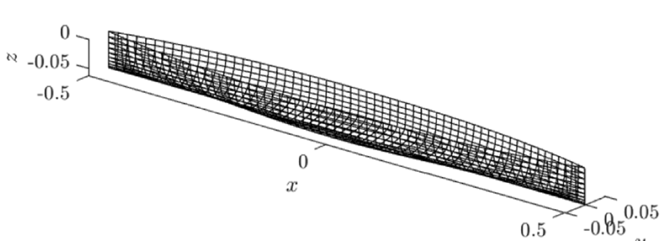

Fig. 1. A panel discretization of a ship-like geometry.

$$G_0 = \frac{1}{r} - \frac{1}{r'} \quad (2)$$

$$\tilde{F}(R, Z, t) = 2 \int_0^\infty e^{kZ} J_0(kR) \sqrt{k}\, \sin(\sqrt{k}\, t) dk \quad (3)$$

with

$$R = \sqrt{(x-\xi)^2 + (y-\eta)^2}, \quad r = \sqrt{R^2 + (z-\zeta)^2} \quad (4)$$

$$Z = z + \zeta, \quad r' = \sqrt{R^2 + Z^2}. \quad (5)$$

The wave part of the Green function, $\tilde{F}(R, Z, t)$, can be written as a function of only two variables by making the substitution $kr' \to \lambda$ to write

$$\tilde{F}(R, Z, t) = \frac{2}{\sqrt{r'^3}} F(\mu, \tau) \quad (6)$$

$$F(\mu, \tau) = \int_0^\infty e^{-\lambda\mu} J_0\left(\lambda\sqrt{1-\mu^2}\right) \sqrt{\lambda}\, \sin(\sqrt{\lambda}\,\tau) d\lambda \quad (7)$$

where

$$\mu = -Z/r', \quad \tau = t/\sqrt{r'}. \quad (8)$$

We note that $0 \le \mu \le 1$, with $\mu = 0$ being the case where both points lie on the free-surface and $\mu = 1$ the case where the two points lie on the same vertical axis ($R=0$).

In Clement (1998), it was shown that $F(\mu, \tau)$ is the solution to the following fourth-order ODE:

$$F^{(4)} + \mu\tau F^{(3)} + \left(\tfrac{1}{4}\tau^2 + 4\mu\right) F^{(2)} + \tfrac{7}{4}\tau F^{(1)} + \tfrac{9}{4} F = 0 \quad (9)$$

with initial conditions

$$F^{(2k)}(\mu, 0) = 0, \quad F^{(2k+1)}(\mu, 0) = (-1)^k (k+1)!\, P_{k+1}(\mu), \quad k = 0, 1, \ldots \quad (10)$$

where P_k are the Legendre polynomials and the notation $F^{(n)} = \partial^n F / \partial\tau^n$ has been adopted. Clement (1998) also derived very similar ODEs for the horizontal and the vertical derivatives of F so that the gradient can be computed by solving two additional ODEs. More recently however Li et al. (2015) showed that these derivatives can be computed directly from the higher τ derivatives of F. Specifically,

$$\frac{\partial \tilde{F}}{\partial Z} = -\frac{2}{\sqrt{r'^5}} F^{(2)} \quad (11)$$

$$\frac{\partial \tilde{F}}{\partial x} = \frac{x-\xi}{R} F_R \quad (12)$$

$$\frac{\partial \tilde{F}}{\partial y} = \frac{y-\eta}{R} F_R \quad (13)$$

where

$$F_R(\mu, \tau) = \frac{1}{\sqrt{1-\mu^2}} \left[\frac{3}{2} F + \frac{\tau}{2} F^{(1)} + \mu F^{(2)}\right]. \quad (14)$$

Thus the Green function and its gradient can all be obtained from the solution to Eq. (9). As noted by Dai (2010), this method can be used for non-zero forward speed problems by tabulating the coefficients in the fixed reference frame and then using interpolation with the arguments shifted to the moving frame. A