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Approximate dynamic programming based control of hyperbolic PDE systems using reduced-order models from method of characteristics

Sudhakar Munusamy, Sridharakumar Narasimhan*, Niket S. Kaisare[1]

Department of Chemical Engineering, Indian Institute of Technology Madras, Chennai 600036, India

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ABSTRACT

Approximate dynamic programming (ADP) is a model based control technique suitable for nonlinear systems. Application of ADP to distributed parameter systems (DPS) which are described by partial differential equations is a computationally intensive task. This problem is addressed in literature by the use of reduced order models which capture the essential dynamics of the system. Order reduction of DPS described by hyperbolic PDEs is a difficult task as such systems exhibit modes of nearly equal energy. The focus of this contribution is ADP based control of systems described by hyperbolic PDEs using reduced order models. Method of characteristics (MOC) is used to obtain reduced order models. This reduced order model is then used in ADP based control for solving the set-point tracking problem. Two case studies involving single and double characteristics are studied. Open loop simulations demonstrate the effectiveness of MOC in reducing the order and the closed loop simulations with ADP based controller indicate the advantage of using these reduced order models.

1. Introduction

Several typical chemical engineering systems such as fixed bed reactors, plug flow reactors, etc. are inherently nonlinear. The dynamic behavior and spatial variation of key state variables are accurately captured by first principle partial differential equation (PDE) models. These PDE models are valid in a larger operating range as compared to lumped parameter ODE models and have been extensively used for design and analysis. It is expected that use of such PDE models in model based controllers will result in improved closed loop performance. However, the use of these models in online control applications is limited. There are two key issues that need to be addressed: the computational effort involved in solving nonlinear optimal control problems and the fact that the state space form of the PDE models is infinite dimensional.

Typical model based controllers such as model predictive control (MPC) require solution of a multi-step optimization problem with a finite horizon cost. An alternative is to formulate a dynamic programming problem which involves the solution of single step optimization problem with an infinite horizon cost. DP employs the 'Bellman principle of optimality' to obtain the 'optimal cost-to-go' values which is the optimal cost involved to reach the set-point starting from current state. This cost-to-go value is obtained for all the points in the discretized state space. Function approximators are used to get the map between cost-to-go values and the points in the state space. The exponential increase in the number of points in state-space as the state dimension increases which is called 'curse of dimensionality' in literature is addressed by formulating an approximate version, viz., ADP by focusing on the relevant state space region traced by other sub-optimal controllers. The cost-to-go values approximate the optimal infinite horizon cost and hence the use of longer horizon can be avoided, thereby reducing the computational load significantly.

In recent years, ADP has been successfully applied to several applications such as a complex bio-reactor characterized by multiple steady states (Kaisare, Lee, & Lee, 2003), integrated plants involving a reactor and distillation column with a recycle (Tosukhowong & Lee, 2009), systems described by partial differential equations (Midhun & Kaisare, 2011; Padhi & Balakrishnan, 2003), etc. However application of ADP to distributed parameters system is a challenging task because of the high computational effort in solving PDEs. In addition, a high order state space representation of DPS results in computational issues in obtaining cost-to-go estimate using function approximators (Lee, Kaisare, & Lee, 2006). This necessitates the use of reduced order models for application of ADP based control to DPS.

Model order reduction techniques generate a finite dimensional state space model that describes the original system dynamics fairly accurately while requiring lower computational effort. There are several order reduction techniques that have been proposed

* Corresponding author. Tel.: +91 44 22574177.
E-mail address: sridharkrn@iitm.ac.in (S. Narasimhan).
[1] Currently at: ABB Corporate Research Center, Bhoruka Tech Park, Whitefield Main Road, Bangalore 560048, India.

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to develop reduced order models suitable for control. Method of lines (MOL) is a conventional order reduction method where the spatial derivative at each nodal point is calculated through appropriate finite difference based approximation (Dochain, Babary, & Tali-Maamar, 1992; Midhun & Kaisare, 2011; Sorensen, Jorgensen, & Clement, 1980). Proper orthogonal decomposition (POD) is another popular method which is suitable for parabolic PDE systems (PDEs involving a second order spatial differential operator) which exhibit modes of different energy. Reduced order models generated using POD have significantly lower dimension compared to those generated by the finite difference method and hence been extensively used in the design of finite dimensional controllers for systems described by parabolic differential equations (Padhi & Balakrishnan, 2003; Pitchaiah & Armaou, 2010; Shvartsman et al., 2000; Shvartsman & Kevrekidis, 1998).

Convection dominated systems such as plug flow reactors, fixed bed reactors, heat exchangers, etc., are described by a set of first order hyperbolic PDEs. The use of finite difference method for such system requires very large number of states to satisfactorily describe the system. Also order reduction through modal decomposition is not possible as the spatial operator of hyperbolic PDE has modes with nearly equal energy (Christofides & Daoutidis, 1998). Hence optimal control techniques which retain the infinite dimensional nature of the systems have been reported (Balas, 1986; Choe & Chang, 1998; Lo, 1973; Wang, 1966). However design and implementation of such infinite dimensional optimal control techniques is complicated. On the other hand, finite dimensional controllers are very well developed and hence there is a need to develop a method which reduces the order of the first order hyperbolic PDEs for design of finite dimensional controllers.

Method of characteristics (MOC) is a conventional solution method for solving first order hyperbolic PDEs and has also been used in the model based control. A combination of MOC with sliding mode control was proposed for nonlinear hyperbolic systems (Hanczyc & Palazoglu, 1995). A output feedback control methodology for systems described by a first order hyperbolic PDE has been developed (Christofides & Daoutidis, 1996). Lyapunov based robust controller was developed for nonlinear hyperbolic PDE (Christofides & Daoutidis, 1998). A combination of finite difference method with MOC has been employed for the control of convection dominated parabolic systems (Shang, Forbes, & Guay, 2007). Finite difference schemes which retain the FIR property of co-current reactor have been proposed and used for the design of controllers (Choi, 2007; Choi & Lee, 2004, 2005). Model predictive control (MPC) based on MOC for fixed bed reactor, plug flow reactor and catalytic flow reversal reactor have been proposed in literature (Fuxman, Forbes, & Hayes, 2007; Mohammadi, Dublijevic, & Forbes, 2010; Shang, Forbes, & Guay, 2004). Though there are applications involving MOC, order reduction using MOC and its subsequent use in model based control is not discussed in the literature. However, the focus of these has been use of MOC as a solution technique, rather than a model order reduction technique. Further in these works, simple approximations such as assuming constant values for the nonlinear terms in the resulting ODEs is employed. In our work we propose an improved method which results in a accurate reduced order model of significantly lower order and complexity.

The primary focus of this article is two-fold. One involves proposing better approximation in the implementation of MOC to hyperbolic PDEs which results in reduced order model. As already mentioned ADP has the advantage of improved closed loop performance with reduced computational load and applying this concept to PDEs systems requires reduced order models. So second part of this work involves applying the reduced order model from MOC in ADP based controller. In the closed loop simulation, we allow for plant-model mismatch by using reduced order model for prediction and optimization and higher order model from MOL as a 'plant'. Though the use of reduced order from MOC in ADP requires interpolation at every sampling instant, we have shown that the resulting closed loop response shows better behavior compared to the nonlinear MPC. To summarize, the contribution of the paper is a new methodology to obtain reduced order models using MOC for systems described by quasi-linear hyperbolic PDEs. Further the robustness of this reduced order model is illustrated through the closed loop simulation involving ADP based control.

The organization of this paper is as follows: Initially the theory behind approximate dynamic programming and the problem in using high dimensional state space models is explained. Next, method of characteristics as a order reduction method is introduced and the methodology of obtaining reduced order models through MOC is explained in detail. This is followed by presentation of the closed loop simulation results using ADP employing MOC for the two case studies involving single and double characteristics. The overall advantages and shortcomings in employing reduced order models from MOC in ADP based control for DPS system are then summarized.

2. Approximate dynamic programming

Consider a system represented by the following nonlinear discrete-time state space model:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) \\ y_k &= g(x_k) \end{aligned} \tag{1}$$

where x_k represents system state, u_k represents manipulated inputs and y_k represents controlled outputs at the kth time instant, respectively. A standard optimal control problem involves minimization of the following stage-wise performance criterion:

$$V_p = \sum_{i=k}^{i=k+p-1} \phi(x_i, u_i) \tag{2}$$

Here, $\phi(x_i, u_i)$ is the single stage cost incurred at state x_i on implementing the control move u_i, and p is the prediction horizon. In the infinite-horizon problem, we let $p=\infty$. A typical single-stage cost in set-point tracking problem is:

$$\phi(x_i, u_i) = (y_{i+1} - y_{sp})^T Q (y_{i+1} - y_{sp}) + \Delta u_i^T R \Delta u_i \tag{3}$$

The optimal infinite-horizon cost-to-go function is defined as

$$\begin{aligned} J^{opt}(x_k) &= \min_{u_k, u_{k+1}, \ldots} V_\infty \\ &= \min_{u_k, u_{k+1}, \ldots} \sum_{i=k}^{i=\infty} \phi(x_i, u_i) \end{aligned} \tag{4}$$

This optimal cost-to-go function satisfies the following Bellman equation:

$$J^{opt}(x_k) = \min_{u_k} [\phi(x_k, u_k) + J^{opt}(x_{k+1})] \tag{5}$$

for all states $x_k \in X$. Note that $J^{opt}(x_{k+1})$ represents the optimal cost-to-go values at the successor state, $x_{k+1} = f(x_k, u_k)$. The computation of $J^{opt}(x)$ that satisfies Eq. (5), either analytically or numerically, is the central crux of dynamic programming (DP). The corresponding optimal input move is computed from the optimal cost-to-go function as:

$$u_k^{opt} = \text{argmin}_{u_k} [\phi(x_k, u_k) + J^{opt}(x_{k+1})] \tag{6}$$

Remark 1. Model predictive control is an open-loop optimal control policy. The finite-horizon optimization problem

$$\text{argmin}_{u_k, \ldots, u_{k+p-1}} V_p(x_k) \tag{7}$$