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Stochastic roll response for a vessel with nonlinear damping models and steady heeling angles in random beam seas

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ABSTRACT

Loss of ship stability is most frequently associated with extreme roll motion. For the case of random beam sea, a single-degree-of-freedom (SDOF) model is applied in the present study in order to represent the stochastic rolling behavior. The random wave excitation term in the SDOF model is approximated as a filtered white noise process by applying a second order linear filter. Accordingly, the original SDOF model is extended into a four-dimensional (4D) dynamic system. The coupled dynamic system can be viewed as a Markov system whose probabilistic properties are governed by the corresponding Fokker–Planck equation. Based on the convenient Markov property, a host of useful response statistics can be obtained by an efficient path integration (PI) method. Different nonlinear damping models, i.e. the linear-plus-quadratic damping (LPQD) model and the linear-plus-cubic damping (LPCD) model, and their effects on the stochastic roll response are investigated and the influence of the steady heeling angle on the response level associated with ship rolling in random seas is also studied. Furthermore, the accuracy of the response statistics computed by the PI technique is verified by means of the versatile Monte Carlo simulation (MCS) technique.

1. Introduction

Excessive rolling motion is considered to be a major cause of stability failure or even capsizing of ships exposed to ocean waves. For large amplitude roll motion, the nonlinear effects associated with the damping and restoring terms have a significant influence on the high-level roll response. Currently, the criteria of the International Maritime Organization (IMO) for evaluation of the intact stability are based on both hydrostatics and dynamics (IMO, 2008). In addition, due to the stochastic nature of the ocean environment and the corresponding wave excitation, the assessment of extreme rolling motion should inevitably be based on dynamic considerations and probabilistic approaches. With the awareness of the deficiencies of the current criteria for intact stability evaluation, the IMO is currently developing the next generation of such criteria with a certain consideration of the physics associated with the dynamics of nonlinear roll motion and the randomness of wave excitation and roll response (Francescutto, 2016). In this work, the dynamic stability is evaluated by means of a probabilistic approach, which may provide insight associated with the nonlinear roll dynamics in random beam seas. Hopefully, this may also serve as a contribution to the second generation IMO intact stability criteria which are currently being developed.

The problem of estimating the stochastic response of such nonlinear dynamic systems excited by random external loads has been a demanding challenge in the past decades. For this type of problem, elaborate theoretical model as well as appropriate mathematical techniques are essential (Ellermann, 2009). In the literature, the roll motion is generally assumed to be decoupled from the other motions and governed by a single-degree-of-freedom (SDOF) model in which the nonlinearities associated with the damping and restoring terms as well as the randomness of the wave excitation are all incorporated (Roberts and Vasta, 2000). Even though the SDOF model is not recommended for actual ship design, it is a very important model for qualitative studies and understanding of the nonlinear behavior under stochastic excitation. The methodology based on the Markov model has been a popular way to analysis the stochastic response of the nonlinear roll motion in random seas (Francescutto and Naito, 2004; Su and Falzarano, 2013; Chai et al., 2015b). Since the Markov model is only valid for a dynamic system driven by white noise or filtered white noise, a second order linear filter is introduced in order to approximate the random wave excitation as a filtered white noise process. Subsequently, the original SDOF model, also a second order differential equation, is extended into a four-dimensional

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(4D) Markov dynamic system. For the coupled system, the probabilistic properties of the roll motion is governed by the Fokker–Planck (FP) equation.

The extended dynamic system usually corresponds to a high-dimensional FP equation, but analytical solutions to high-dimensional FP equations are only available for some linear systems and a very restricted class of nonlinear systems. As for numerical solutions, the path integration (PI) method has been proved to be an efficient approximation for solving the high-dimensional FP equations. This method is based on the Markov property of the dynamic system and the global solution, i.e., the evolution of the response statistics, is calculated by linking the explicitly known local solutions via a step-by-step solution technique (Mo, 2008). In addition to its efficiency, the high-dimensional PI method is able to provide reliable estimation of the response statistics, even for high response levels which are associated with low probabilities (Naess and Johnsen, 1993; Karlsen, 2006; Chai et al., 2015b).

As an alternative to the efficient PI method, Monte Carlo simulation (MCS) is another approach that can be applied in order to determine the response statistics of nonlinear systems subjected to random external or parametric forcing. The nonlinear and time-dependent terms in the dynamic system can then be easily dealt with. However, the MCS approach is only a brute force alternative and the associated computational efficiency will be sacrificed for estimation of extreme responses with low probability levels. In this work, the straightforward MCS method will serve as a verification tool to evaluate the response statistics obtained by the efficient PI method.

The nonlinearity of the roll damping moment has been recognized to be crucial for evaluating ship stability since Froude's time, however, the dynamic effects of the damping term have not been taken into consideration in the current intact stability criteria. Moreover, for traditional ship motion analysis and the associated reliability-based design, the damping and restoring terms are usually linearized for simplicity (Bulian and Francescutto, 2004). Generally, the damping moment has three kinds of components: the damping caused by radiation at the free surface and the damping caused by vortex shedding and flow separation as well as the viscous friction damping. Since these terms are coupled with each other, the quantitative evaluation of roll damping moment is difficult and empirical models are applied to describe the roll damping term. The linear-plus-quadratic damping (LPQD) model, which has been verified by numerous studies of experimental data, is widely used to describe the damping term in the SDOF model for the roll motion (Roberts and Vasta, 2000). On the other hand, the LPQD model is only once continuously differentiable, while another empirical damping model, the linear-plus-cubic damping (LPCD) model is infinitely differentiable and mathematically preferable to the LPQD model. In this work, the LPQD model is approximated by the LPCD model with the assistance of a numerical procedure proposed by Bikdash et al. (1994) and the effect of different damping models (i.e., the equivalent LPQD and LPCD models) on the stochastic roll response, especially on the high level response will be studied.

In practice, it has been observed that the stability properties of vessels with steady heeling (i.e. roll bias) are worse than those which correspond to upright conditions, i.e., vessels without such a steady heeling angle. However, current intact stability criteria only consider the influence of the heeling moment on the ship stability in a hydrostatic manner. Generally, heeling moments can be caused by wind load, by transverse displacements of masses or by lateral pull during towing work, etc. (Biran and Pulido, 2013). As for the most common mean wind action, the heeling moment is proportional to the square of the wind velocity and it also depends on the lateral projected area of the above-sea surface part of the hull. The performance of a "biased vessel" in random beam seas has been studied by stochastic linearization (Bulian and Francescutto, 2004) and the Melnikov criterion (Jiang et al., 2000, 1996), etc. Up to now, there seems to be no investigation with respect to the effect of a steady heeling moment on the distribution of random roll motion as well as on the stochastic roll response. By application of the efficient PI method, the above effects can be investigated directly since a host of useful response statistics can be obtained by solving the corresponding FP equation.

The present paper is organized as follows. Section 2 describes the SDOF model for the roll motion and the linear filter technique used to approximate the random wave excitation is also presented. The principle and numerical implementation of the efficient PI method are described in Section 3. Results from numerical simulations are presented in Section 4 where the response statistics obtained by the conventional MCS technique are also given. Furthermore, in this Section, the influence of different damping models and steady heeling angles on the stochastic roll response are also illustrated.

2. Physical modeling

2.1. Mathematical model of roll motion

By neglecting the coupling with other modes of motion, the rolling behavior of the vessel in random beam seas can be represented by the following SDOF model for qualitative study (Roberts and Vasta, 2000):

$$(I_{44} + A_{44})\ddot{\phi}(t) + B(\dot{\phi}(t)) + \Delta(C_1\phi(t) - C_3\phi^3(t)) = M(t) \tag{1}$$

where $\phi(t)$ and $\dot{\phi}(t)$ are the roll angle and the roll velocity, respectively. I_{44} is the moment of inertia, A_{44} denotes the added mass coefficient. $B(\dot{\phi}(t))$ is the damping moment term and $\Delta(C_1\phi(t) - C_3\phi^3(t))$ is the restoring moment term. Δ is the displacement of the vessel, C_1 and C_3 are the linear and nonlinear roll restoring coefficients of the restoring arm, respectively. $M(t)$ represents the random wave excitation moment due to external waves.

It should be noted that the roll motion has a softening characteristic since the nonlinear stiffness term is negative. For the softening cases, ship capsizing would occur when the roll angle exceeds the angle of vanishing stability beyond which the restoring moment becomes negative.

The roll excitation moment is assumed to be a stationary Gaussian process and it can be characterized by the roll excitation moment spectrum, $S_{MM}(\omega)$. The latter is related to the wave energy spectrum, $S_{\xi\xi}(\omega)$, by the following relationship:

$$S_{MM}(\omega) = |F_{roll}(\omega)|^2 S_{\xi\xi}(\omega) \tag{2}$$

where $|F_{roll}(\omega)|$ represents the roll moment amplitude per unit wave height at frequency ω which can be obtained e.g. by application of strip theory. The wave elevation process is governed by the wave energy spectrum and it is also assumed to be a stationary Gaussian process during a short-term period.

The LPQD model is widely used to describe the damping moment term, and the empirical model is expressed as:

$$B(\dot{\phi}(t)) = B_{44l}\dot{\phi}(t) + B_{44q}\dot{\phi}(t)|\dot{\phi}(t)| \tag{3}$$

in which, B_{44l} and B_{44q} are the linear and quadratic damping coefficients for the LPQD model, respectively.

However, the LPQD is only once continuously differentiable and this is not very appealing for analytical treatment e.g. by application of the perturbation method, bifurcation analysis and so on. The LPCD model is infinitely differentiable and it is often applied in order to approximate the LPQD model by the following expression: