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# Ship dynamic stability assessment based on realistic wave group excitations

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ABSTRACT

Ship propensity for stability failure in irregular beam seas is addressed. A new method, based on the combination of the Karhunen–Loève theorem with the theory of Markov processes, is employed in order to construct realistic wave groups of varying heights and periods. The focus is set on waveforms with high probability of occurrence, given a sea-state. These waveforms are then applied, as beam-sea excitation, to a modern container vessel. Safe basin integrity diagrams and transient capsize diagrams are produced through short numerical simulations. For comparison purposes, alternative forms of excitation, such as wave groups of varying heights but fixed periods; and regular wave groups of fixed heights and periods, are also considered. Differences in the system's transient response, specifically on the integrity of its safe basin and on the transient capsize diagram, are discussed. The proposed method advances the so called "critical wave groups method" and it can be used for a swift assessment of ship stability.



## 1. Introduction

The study of large amplitude ship motions in a stochastic sea is one of the most challenging computational tasks in naval architecture. On the one hand, advanced methods of nonlinear dynamics are indispensable for yielding insights into the mechanisms of capsizing. At the same time, however, such methods have not been so practical, yet, in providing estimates of capsizing tendency, especially when employing computationally expensive numerical models. This is compounded by the fact that, for quantitative accuracy in dynamic stability predictions, detailed hydrodynamic modelling is highly desirable. For rare phenomena like capsizing, the efficiency of performing long-time simulations on heavy models is disputable, since most of the time is idly expended on simulating innocuous ship-wave encounters. This has motivated the development of a number of techniques for directly extracting those time intervals when hazardous encounters occur.

A relevant phenomenon, often observed in wind-generated seas, is wave grouping. Wave groups are sequences of high waves with periods varying within a potentially small range (Masson and Chandler, 1993; Ochi, 1998). Notably, the occurrence of dangerous wave group events, leading to motion augmentation, does not necessarily imply exceptionally high waves. Resonant phenomena, often "felt" in the first few cycles of wave group excitation, are crucial for the integrity of a marine system. The assessment of ship stability under the effect of wave groups was the subject of three recent studies, reviewed, in brief, next.

Reaping benefit from a separation of ship dynamics from wave field randomness, the "critical wave groups" approach disassembles the problem in a deterministic and a probabilistic part (Themelis and Spyrou, 2007). In the former, critical combinations of heights, periods and run lengths, associated with regular wave groups that incur unacceptably large dynamic response, are identified. The critical, in terms of ship stability, waveforms represent basically thresholds, defined by regular wave trains. Statistical analysis of the seaway is included in the "probabilistic" part of the approach. The propensity for ship stability failure is quantified by calculating the probability of encountering any group higher than the determined critical one.

A statistical approach for the prediction of extreme parametric roll responses was presented by Kim and Troesch (2013). In this method it was assumed that the fluctuation of the instantaneous *GM* is a Gaussian random process. A "Design Load Generator" was then employed in order to generate an ensemble of irregular wave groups, associated with the extreme value distribution of a surrogate process, representing time-varying metacentric height groups (Kim, 2012). The derived wave trains, realized as a lower bound of the "true" excitation, were, eventually, utilised as input to a hydrodynamic model for simulating the actual nonlinear response of a C11 class containership.

Malara et al. (2014) proposed an approach for the estimation of the maximum roll angle, induced by spectrum-compatible wave group excitation. Representation of the load process in the vicinity of an exceptionally high wave was formulated within the context

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of the "Quasi-Determinism" theory (Boccotti, 2000). The approach is asymptotically valid in the limit of infinitely high waves and its use is possibly suitable for heights at least two times the significant wave height of the considered sea state (Boccotti, 2000).

The objective of the current work is to develop a practical probabilistic method of stability assessment based on the examination of the effect of a series of short duration wave group excitations, of realistic form, on a ship's transient response. Stability performance will be evaluated by comparing the "engineering integrity" and the "transient capsize diagrams" (Rainey and Thompson, 1991), obtained by the current method, versus what would have been obtained if the "critical wave groups" method had been applied.

In the following section, a spectrum compatible method of wave group construction will be proposed. The method expands upon Themelis and Spyrou (2007) on the one hand, by introducing realistic wave group profiles; and on Malara et al. (2014), by removing the "extreme waves" assumption imposed by the theory of Quasi-Determinism.

## 2. Wave group modelling

### 2.1. Stochastic treatment of wave successions

The assumption of height sequences which fulfil the Markov property has been employed with remarkable success in a number of studies for the derivation of wave groupiness measures (Kimura, 1980; Battjes and van Vledder, 1984; Longuet-Higgins, 1984). On the other hand, the application of straightforward spectral techniques, targeting the statistical elaboration of wave period groupings, is full of inherent limitations.

Recently, an extended Markov-chain model, allowing for cross-correlations between successive heights and periods, was proposed (Anastopoulos et al., 2015). The joint expectations of consecutive heights $\overline{h}_i$ and periods $\overline{t}_i$ were expressed by the following set of coupled equations:

$$\overline{h_i} = \int_0^{\infty} h_i f_{H_i|H_{i-1},T_{i-1}}(h_i|h_{i-1}, t_{i-1})\mathrm{d}h_i \tag{1a}$$

$$\overline{t_i} = \int_0^{\infty} t_i f_{T_i|H_{i-1},T_{i-1}}(t_i|h_{i-1}, t_{i-1})\mathrm{d}t_i \tag{1b}$$

where $H_i$ and $T_i$ are random processes with state variables $h$ and $t$, respectively, referring to the height and period of a single wave at time step $i$. The kernels, $f_{H_i|H_{i-1},T_{i-1}}$ and $f_{T_i|H_{i-1},T_{i-1}}$, are the transition probability densities of the process.

The current calculation scheme is focused on producing the "most expected" wave successions, given a predefined set of values for the height and period of the highest wave of the group. These values are used for initiating the iterative scheme established through Eqs. (1a) and (1b). In detail, forward application of the iteration determines the heights and periods of the waves succeeding the initial (highest) one, that are most likely to be realized. At the same time, the "time-reversibility property" of the Markovian system justifies the application of the same procedure backwards in time so as to identify the "most expected" past outcomes. As realized, the results depend explicitly on the height $H_c$ and period $T_c$ of a reference wave, which is located at the centre of the generated sequence being also the highest. Eventually, the process is terminated once we observe a wave height lower than a specified threshold value $H_{cr}$. Then, we compute the group length $j$, which is equal to the total number of predicted heights greater than $H_{cr}$ (see e.g., Masson and Chandler (1993); Ochi (1998)).

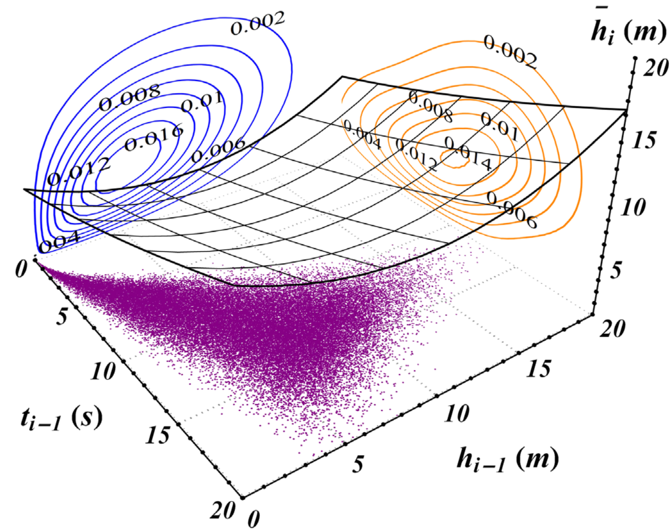


**Fig. 1.** Correlation surface for the prediction of the "most expected" wave heights.

### 2.2. Calculation of the transition kernels

A direct method to obtain the transition probabilities of Eqs. (1a) and (1b) is through Monte Carlo simulations of the water surface elevation. It is noted however that, in a parallel development, a computational method, based on envelope analysis in conjunction with the theory of copula distributions, could produce them, with reasonable accuracy, in the form of explicit formulas (Anastopoulos et al., 2015).

The key for the derivation of the transition probabilities, in the current work, is to extract the heights and periods from the generated records; arrange them in the vector sets of Eq. (2) and, finally, perform regression analysis. Then, the transition mechanisms can be expressed through a best-model-fit method.

$$\mathbf{A} = \begin{bmatrix} h_i \\ h_{i-1} \\ t_{i-1} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} t_i \\ h_{i-1} \\ t_{i-1} \end{bmatrix} \tag{2}$$

In Fig. 1, the concept of a "correlation surface", which fits the data of vector **A**, is explained. In the same figure, the $(h_{i-1} - t_{i-1})$ plane corresponds to the total population of joint height-period realisations. The smoothened bivariate height and joint height-period distributions for successive waves are also provided.

For the simulations of the surface displacement, a JONSWAP spectrum (Hasselmann et al., 1973), with peak period $T_p = 13.6$s and significant wave height $H_s = 10$m, was selected.

### 2.3. The Karhunen–Loève representation

The Karhunen–Loève theorem is invoked in order to construct continuous-time analogues for the "most expected" height and period successions following from the application of the Markov-chain model described in the above (Karhunen, 1947; Loève, 1978). The advantage of the specific approach over the traditional Fourier series representation is that it ensures the minimum total mean-square error resulting out of its truncation. The theorem states that the water surface elevation $\eta$ admits the following decomposition over a finite time interval $(-T, T)$:

$$\eta(t) = \sum_{n=0}^{\infty} a_n f_n(t) \tag{3}$$

In the case of a Gaussian random process, the coefficients $a_n (n = 0, 1, \ldots)$ are random independent variables. The