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The optimal design of a flap gate array in front of a straight vertical wall: Resonance of the natural modes and enhancement of the exciting torque

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We consider a wave energy converter made of an array of Q neighbouring floating flap gates with finite thickness in front of a straight vertical wall in constant depth. Solutions of the radiation and scattering problems are achieved by application of Green's theorem and Green's function yielding a system of hypersingular integral equations for the velocity potential expanded in terms of Legendre polynomials. We investigate how the distance between the array and the vertical wall affects the performance of the array under the action of monochromatic and random waves. We show that large values of the exciting torque on the gates can be obtained by tuning the wall distance with the resonance of the natural modes of the array; this in turn yields large power extraction for a wide range of frequencies.

1. Introduction

Research on wave energy converters (WECs) has undergone a remarkable development in recent years. Lots of attention have been given to the individuation of the optimal layout which maximizes power absorption. Lately, the oscillating wave surge converter (OWSC) has been proven very effective (Babarit et al., 2012; Renzi et al., 2014; Whittaker and Folley, 2012). Analytical and numerical models have been developed to analyse the behaviour of a single OWSC in a channel (Renzi and Dias, 2012) and in open sea (Renzi and Dias, 2013). The occurrence of resonant amplification due to interaction of multiple floating bodies through wave motion has been investigated in Adamo and Mei (2005), Li and Mei (2003), Falnes and Hals (2012), Panizzo et al. (2006), Sammarco et al. (1997), Sammarco et al. (1997), Srokosz and Evans (1979), and Thomas and Evans (1979). Consistently, the analysis has been extended to systems of multiple OWSCs in which several piercing gates interact (Michele et al., 2015; Renzi and Dias, 2014; Renzi et al., 2014; Sammarco et al., 2013; Sarkar et al., 2014). Works have focused also on investigating the behaviour of a single wave energy converter near a reflecting vertical wall (Evans, 1988; Evans and Porter, 1995, 1996; Lovas et al., 2010; Martins-Rivas and Mei, 2009; Sarkar et al., 2015). Evans (1988) solved the problem of a point absorber in front of a vertical wall. He found that for certain values of the distance between the wall and the point absorber, the capture width (Mei et al., 2005) reaches large values. Similar results have been obtained by Sarkar et al. (2015) for a flap-type oscillating wave energy converter with small thickness near a straight coast. Evans and Porter (1996) solved the two-dimensional problem of a thin rolling plate next to a vertical wall. They found that strong resonance occurs at the frequencies of the sloshing modes. However the effects of a straight vertical wall on the behaviour of a system of OWSC's have not been investigated yet.

In this paper, the behaviour of a single array made of Q floating flap gates with finite thickness in front of a straight vertical wall is analysed. We extend the solution of Michele et al. (2015) for the radiation and scattering problems in terms of Green's function and hypersingular integral equations. In order to account for the no-flux boundary condition at the wall, the Green function has been modified by making use of the method of images (Linton and McIver, 2001; Morse and Feshbach, 1981). The complexity of the boundaries is then reduced and the mathematical problem considerably simplified. We show that the array in front of a reflecting wall achieves larger values of the capture factor with respect to both the case of a gate farm in open sea (Michele et al., 2015) and a single gate in front of a reflecting wall (Sarkar et al., 2015). Indeed, such a system benefits from the mutual interaction between the resonance of the natural modes and from the wall induced enhancement of the exciting torque. A parametric analysis in monochromatic incident waves reveals also to what extent the distance between array and vertical wall can affect the efficiency of the device. Finally, the behaviour and performance of the array

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under the action of incident waves represented by the JONSWAP spectrum is investigated and discussed (Hasselmann et al., 1973; Goda, 2012; Eriksson et al., 2005; Sarkar et al., 2013).

2. Governing equations for Q gates

Referring to Fig. 1, consider a finite single array made of Q identical flap gates in front of a reflecting vertical wall. Let h be the constant water depth and L the distance between the array and the wall. a and $2b$ are respectively the width and the thickness of each gate, hence $w=aQ$ represents the total width of the array. Define a three-dimensional Cartesian coordinate system with the x and y axes lying on the mean free surface and the z axis pointing vertically upward. The y axis is directed along the wall which spans from $y=-\infty$ to $y=\infty$, while the x axis is orthogonal both to the array and to the vertical wall. All the gates are hinged to a bottom foundation and oscillate about the horizontal common axis lying on $x=L$, $z=-h+c$. The symbol G_q denotes the q-th flap gate. Monochromatic waves of amplitude A, period T and angular frequency $\omega=2\pi/T$ come from $x=+\infty$ and are normally incident on the flaps. Let $\Theta_q(t)$ be the angular displacement of G_q, positive clockwise and define $\Theta(y,t)$ as the angular displacement function of the array:

$$\Theta(y,t)=\{\Theta_1(t),\ \ldots,\ \Theta_q(t),\ \ldots,\ \Theta_Q(t)\}. \tag{1}$$

$\Theta(y,t)$ is an unknown piece-wise function of y. The fluid is considered inviscid and incompressible and the flow irrotational, hence there exist a velocity potential $\Phi(x,y,z,t)$ satisfying the Laplace equation in the fluid domain Ω:

$$\nabla^2\Phi=0,\quad (x,y,z)\in\Omega. \tag{2}$$

On the basis of linearised water-wave theory the potential $\Phi(x,y,z,t)$ satisfies

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0,\quad z=0, \tag{3}$$

and

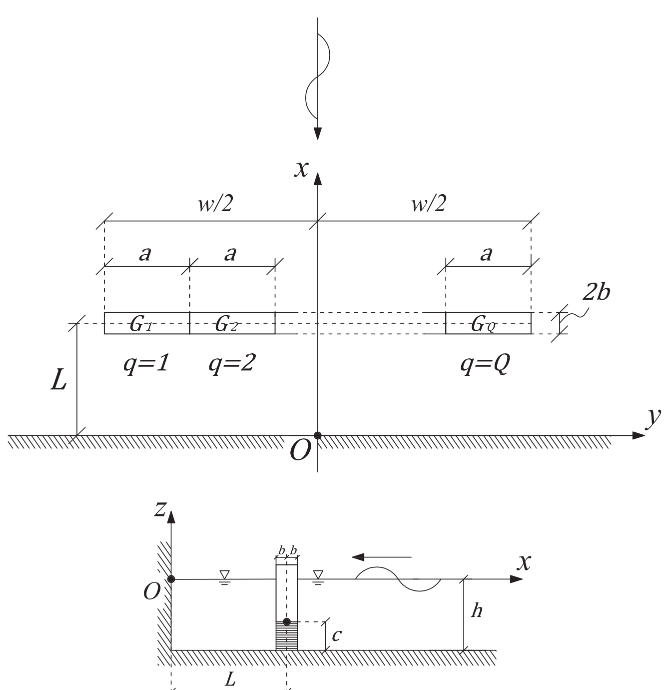

Fig. 1. Plan geometry and side view.

$$\frac{\partial\Phi}{\partial z}=0,\quad z=-h. \tag{4}$$

In the limit of small-amplitude oscillations, the kinematic condition on the array may be written:

$$\frac{\partial\Phi}{\partial x}=\frac{d\Theta_p}{dt}(z+h-c)H(z+h-c),$$
$$x=L\pm b,\quad y\in\left(-\frac{w}{2},\frac{w}{2}\right),\quad z\in\left[-h,0\right], \tag{5}$$

$$\frac{\partial\Phi}{\partial y}=0,\quad x\in(L-b,L+b),\quad y=\pm\frac{w}{2},\quad z\in\left[-h,0\right]. \tag{6}$$

where H is the Heaviside step function, while the kinematic condition on the vertical wall gives:

$$\frac{\partial\Phi}{\partial x}=0,\quad x=0,\quad z\in\left[-h,0\right]. \tag{7}$$

Assuming harmonic fluid and gates motion with frequency ω, the time dependence can be separated as follows:

$$\Phi(x,y,z,t)=\mathrm{Re}\{\phi(x,y,z)e^{-\mathrm{i}\omega t}\}, \tag{8}$$

$$\Theta_q(y,t)=\mathrm{Re}\{\theta_q(y)e^{-\mathrm{i}\omega t}\}. \tag{9}$$

The problem is linear, hence the spatial potential $\phi(x,y,z)$ can be decomposed as:

$$\phi=\phi^I+\phi^F+\phi^S+\sum_{q=1}^{Q}\phi_q^R, \tag{10}$$

where:

$$\phi^I=-\frac{\mathrm{i}Ag}{\omega}\frac{\mathrm{ch}\ k_0(h+z)}{\mathrm{ch}\ k_0h}e^{-\mathrm{i}k_0x}, \tag{11}$$

is the potential of the plane incident waves incoming from $x=+\infty$,

$$\phi^F=-\frac{\mathrm{i}Ag}{\omega}\frac{\mathrm{ch}\ k_0(h+z)}{\mathrm{ch}\ k_0h}e^{\mathrm{i}k_0x}, \tag{12}$$

is the potential of the waves reflected by the vertical wall, ϕ^S is the potential of the scattered waves by the array and ϕ_q^R is the potential of the radiated waves due to the moving gate G_q while all the other gates are at rest. In (11) and (12) k_0 is the wavenumber, i.e. the real root of the dispersion relation $\omega^2=gk_0\ \mathrm{th}\ k_0h$ and i is the imaginary unit. Both ϕ_q^R and ϕ^S must satisfy Laplace equation (2), the mixed boundary condition on the free surface (3) the no-flux condition on the seabed (4) and the no-flux condition on the vertical wall (7).

Let $x^{\pm}$ indicate the x-coordinate of the rest position of the vertical surface of the gates:

$$x^{\pm}=L\pm b. \tag{13}$$

Each gate G_q spans a y-width given by:

$$y\in[y_q,y_{q+1}],\quad y_q=(q-1)a-\frac{w}{2},\qquad q=1,\ \ldots,\ Q. \tag{14}$$

Define the horizontal boundary S_q of the gate G_q as

$$S_q=\left\{x=x^{\pm},\ y\in[y_q,y_{q+1}]\right\}, \tag{15}$$

and the end horizontal boundaries of the array

$$S_w=\left\{x\in(x^-,x^+),\ y=\pm\frac{w}{2}\right\}. \tag{16}$$

The kinematic boundary conditions on the array surfaces and on the vertical wall can be written as follows: