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Free transverse vibration of mono-piled ocean tower

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ABSTRACT

The free vibration of a continuous, elastic model of a mono-piled ocean tower is studied. The tower, partially submerged in water, is undergoing free transverse vibration in a plane. It is modeled as a non-uniform Timoshenko beam which has an eccentric tip mass on one end and is supported by a mono-piled foundation on the other. Effects of shear deformation and rotary inertia are included in the beam. The problem of interaction of soil with the pile is solved by adopting the Winkler Foundation model. The viscoelastic character of the soil is included using the Kelvin–Voigt model. Equation of motion of the tower is derived using Hamilton's variational principle and approximate analytical solution is found using the Rayleigh Ritz Method (RRM). The solution is compared with that obtained from a conventional Finite Element Method (FEM) which shows a good agreement. The trial function of RRM is assumed as uniform beam mode-shapes satisfying the boundary and continuity conditions of the ocean tower. At the end, an extensive parametric study is carried out which provides an insight into the dependence of natural frequency on different configurations of the tower.

1. Introduction

The dynamic behavior of structures like mono-piled ocean tower is an area of extensive research, since they are widely used to support superstructures like wind turbine, offshore platforms etc. It is well known that these ocean towers are often subjected to dynamic loads such as wind, waves, earthquakes etc. It is therefore necessary to calculate natural frequency as well as associated mode-shape to get the preliminary estimation of dynamic behavior of these structures. The upper layer of the seabed consists of soft soil which tends to affect their response. Therefore, it is also important to analyze the interaction of piles with soil in order to have a good estimation of the dynamic behavior of these structures.

There is much literature available which provide the analytical and numerical solutions for evaluating dynamic behavior of such structures. For example, Ankit and Datta (2015) solved the transverse vibration of wet ocean tower by modeling it as non-uniform Timoshenko beam with eccentric tip mass. The foundation was modeled as combination of transverse and rotational spring and dampers. Wu and Hsu (2007) analyzed the free vibration of partially wet, elastically supported uniform Euler-Bernoulli beam with eccentric tip mass using analytical formulation. Uscilowska and Kolodziej (1998) provided closed form solution for a partially immersed cantilever beam with eccentric tip mass. Auciello and Ercolano (2004) provided solution for the non-uniform Timoshenko beam for the free vibration by the energy method. Wu and Chen (2005) solved the free vibration of non-uniform partially wet Euler-Bernoulli beam with elastic foundation and tip mass. De Rosa et al. (2013) calculated closed form solution for free vibration of a linearly tapered, partially immersed, elastically supported column (Euler-Bernoulli beam) with eccentric tip mass. Ece et al. (2007) investigated the vibration of an isotropic beam which has a variable cross-section. Pacheco et al. (2008) presented a model to represent the soil surrounding the pile that includes its inertial effects and investigated their importance on the dynamic response of single piles.

The vibration analysis of mono-piled ocean tower modeled as non-uniform Timoshenko beam has been rarely investigated as the authors know. In the present paper, the ocean tower is modeled as a partially submerged, non-uniform Timoshenko beam having a rigid tip mass with eccentricity at the free end, and non-classical pile foundation at the other end. The pile foundation has been modeled as a distributed spring system which is also known as Winkler Foundation model. The damping effect in the pile–soil interaction is included by using the Kelvin–Voigt model. With the consideration of Timoshenko beam, the effect of shear deformation and rotary inertia is included. The free vibration equation of motion is derived using Hamilton's variational principle and the solution is found using the Rayleigh Ritz Method (RRM) and the Finite Element Method (FEM).

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Nomenclature

x	space variable along the beam length.
t	time variable
u	transverse deflection of non-uniform beam
θ	pure bending slope of non-uniform beam
q	principle coordinate of non-uniform beam
ω	natural frequency of non-uniform beam
ω_R	damping part of ω
ω_I	oscillatory part of ω
L	length of the beam without tip-mass above seabed
α	submergence ratio
β	tapering ratio
δ_1	soft-soil depth ratio
δ_2	pile depth ratio
ρ	density of Steel
ρ_w	density of Water
ρ_{s1}	density of soft-soil
ρ_{s2}	density of stiff-soil
M	bending moment in non-uniform beam
V	shear force in non-uniform beam
A	section area of non-uniform beam
I	sectional area 2nd moment of non-uniform beam
C_A	added mass coefficient
k_{t1}	translational spring constant of soft-soil
k_{t2}	translational spring constant of stiff-soil
c_{t1}	translational damping constant of soft-soil
c_{t2}	translational damping constant of stiff-soil
η_t	proportionality constant of c_{t1} and c_{t2}
E	modulus of elasticity of the material
G	shear modulus
k	shape factor of the cross-section
m_p	tip mass
I_p	rotary inertia of tip mass
r_p	radius of gyration of tip mass
e_p	eccentricity of tip mass
ν	Poisson's ratio
u_F	transverse deflection of uniform beam
θ_F	pure bending slope of uniform beam
q_F	principle coordinate of uniform beam
ω_F	natural frequency of uniform beam
$\omega_{F,R}$	damping part of ω_F
$\omega_{F,I}$	oscillatory part of ω_F
M_F	bending moment in uniform beam
V_F	shear force in uniform beam
A_F	section area of uniform beam
I_F	sectional area 2nd moment of uniform beam
g	gravitational acceleration
φ	uniform beam mode-shape (Trial function)
ϕ	non-uniform mode-shape
ψ	uniform pure bending slope mode-shape (Trial function)
Ψ	non-uniform pure bending slope mode-shape
U	potential Energy
T	kinetic energy
R	Rayleigh dissipation factor
$\mathbf{M}$	mass matrix
$\mathbf{C}$	damping matrix
$\mathbf{K}$	stiffness matrix
ξ	local space coordinate
L_e	length of the beam element
U^e	potential Energy of beam element
T^e	kinetic energy of beam element
R^e	Rayleigh dissipation factor of beam element
W_g^e	work done due to gravity on beam element
n_e	Nth element of the beam
N_e	total number of beam elements

In RRM, a trial function, which is used to obtain non-uniform beam mode-shapes of ocean tower, is assumed as uniform beam mode-shapes satisfying the boundary conditions of ocean tower. In FEM, the Mindlin-type linear beam element of C^0-order with four degrees of freedom, as explained in Bathe (1996), has been used as shape function. This linear beam model is simple to code and hence used here for the verification of results.

The Hamilton's variational principle involves an integral equation and hence, higher order non-uniformity in section area of beam can be handed easily. In both the approaches, i.e., RRM and FEM, the free vibration equation is derived by using this methodology. The difference between RRM and FEM lies in choosing the shape function for finding the solution. In RRM, the shape function is chosen over the entire beam while in FEM, it is

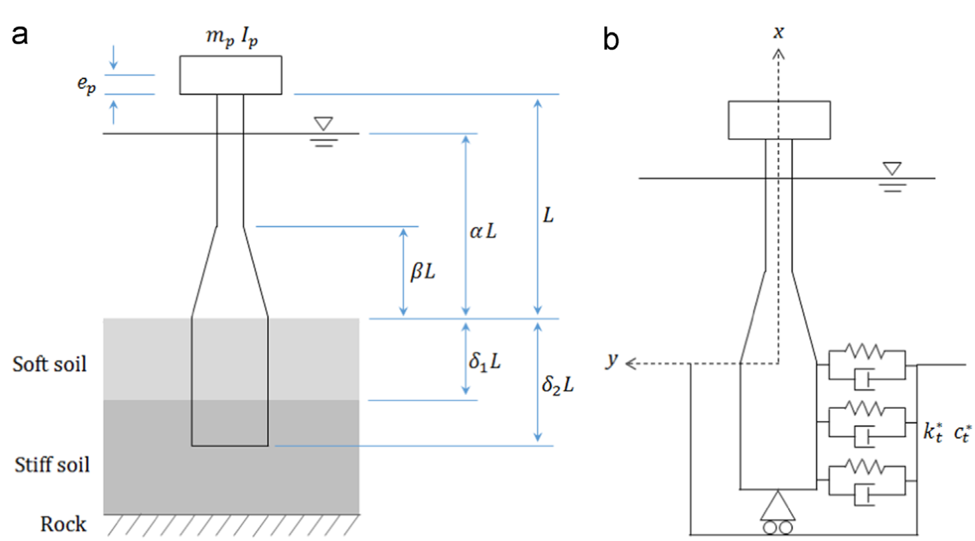

Fig. 1. (a) Ocean Tower model. (b) Mathematical model of the ocean tower.