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Hybrid smoothed finite element method for two-dimensional underwater acoustic scattering problems

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ABSTRACT

It is well known that the standard finite element method (FEM) is unreliable to solve acoustic problems governed by the Helmholtz equation with large wave numbers due to the “overly-stiff” nature of the FEM. In order to overcome this shortcoming, the hybrid smoothed finite element method (HS-FEM) using triangular elements is presented for the two-dimensional underwater acoustic scattering problems. In the HS-FEM, a scale factor $\alpha \in [0, 1]$ is introduced to establish the area-weighted gradient field that contains contributions from both the standard FEM and the node-based smoothed finite element method (NS-FEM). The HS-FEM can provide a close-to-exact stiffness of the continuous system, thus the numerical dispersion error can be significantly decreased. To handle the underwater acoustic scattering problems in an infinite fluid medium, the bounded computational domain is obtained by introducing an artificial boundary on which the Dirichlet-to-Neumann (DtN) condition is imposed. Several numerical examples are investigated and the results showed that HS-FEM can provide more accurate solutions than the standard FEM. Therefore, the present method can be applied to practical underwater acoustic scattering problems such as sonar mine-hunting and sonar detection in ocean acoustics.

1. Introduction

In the past several decades, the time-harmonic acoustic problems in a homogeneous medium governed by the Helmholtz equation have been a very active research field. In general, the acoustic problems can be classified into interior and exterior problems. Duo to limitations of the analytical method, the analytical solutions are only available for very simple geometries. Thus the numerical methods are of great importance in solving these acoustic problems with complex geometries.

One popular and powerful numerical method for coping with the time-harmonic acoustic problems is the standard finite element method (FEM). There is a great deal of relevant research work which can be found in the published literatures (Crocker, 1975; Petyt et al., 1976; Nefske et al., 1982; Harari and Magoulès, 2004). However, the numerical approaches for handling the acoustic problems still remain two major challenges. The first challenge is how to treat the exterior acoustic problems in unbounded domains effectively. Initially, the FEM was applied to acoustics with the aim of solving the interior problems in bounded domains. For exterior problems, including acoustic radiation and scattering, the well-known Sommerfeld radiation condition should be satisfied so that there is no spurious wave reflected from the far field. Another challenge is the numerical dispersion issue (Ihlenburg and Babuška, 1995, 1997; Ihlenburg et al., 1997; Deraemaeker et al., 1999). In general, the numerical methods can obtain relatively accurate results in the low frequency range. However, with the increase of the frequency, the numerical dispersion error will increase dramatically.

In order to use the FEM for exterior acoustic problems, the unbounded domain is usually truncated by an artificial boundary on which the non-reflecting boundary condition is imposed to replace the Sommerfeld radiation condition at infinity. In recent years, a series of numerical treatments including absorbing boundary conditions (Engquist and Majda, 1977; Clayton and Engquist, 1977; Higdon, 1987; Hu, 2004), Dirichlet to Neumann (DtN) boundary conditions (Keller and Givoli, 1989; Harari et al., 1998; Grote and Kirsch, 2004; Givoli et al., 1998) and perfectly matched layer (PML) (Hastings et al., 1996; Berenger, 1994; Turkel and Yefet, 1998) have been proposed to handle exterior acoustic problems. Among them the DtN boundary condition devised by Givoli and Keller is an exact non-reflecting boundary condition. It relates the “Dirichlet datum” to the “Neumann datum” with the

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help of an integral operator. Although this boundary condition is non-local, it still possesses high computational efficiency and can obtain much more accurate results than those obtained from various approximate local conditions. Therefore, many researchers have applied the DtN boundary conditions to cope with all kinds of exterior acoustic problems and other problems in large finite domains.

For the purpose of reducing the numerical dispersion error in acoustic problems, various numerical techniques based on the standard finite element method have been developed to tackle this issue, such as the Galerkin/least-squares finite element method (GLS) (Harari and Hughes, 1992; Harari and Nogueira,2002; Thompson and Pinsky, 1995), the quasi-stabilized finite element method (QSFEM) (Deraemaeker et al., 1999) and the residual-free finite element method (RFEM) (Franca et al., 1997). However, all of the above methods can not reduce the numerical dispersion error effectively.

In addition to the standard finite element method and the extended finite element method, the meshfree methods have been also introduced to solve the acoustic problems. Belytschko et al. (1994) proposed the element-free Galerkin method (EFGM) to tackle the numerical dispersion in acoustic problems. Bouillard and Suleaub (2008) found that the EFGM is effective to reduce the numerical dispersion error significantly compared to the FEM even though the EFGM also suffers from the dispersion and pollution effect. However, in order to control the numerical dispersion error, delicate background cells and a large number of quadrature points are needed for the global numerical integration, leading to prohibitive computational demands. As mentioned in He et al. (2009), the approximate discrete model is the main reason to cause dispersion error. The stiffness of the discretized model obtained from the standard FEM always behaves stiffer than the original model, leading to the so-called numerical dispersion error. So producing a properly "softened" stiffness for the discrete model is much more essential to control the numerical error. Recently, Liu et al. (2007a, 2007b, 2009) have proposed a series of smoothed finite element methods (S-FEM) which are formulated by incorporating the strain smoothing techniques of meshfree methods into the existing standard finite element methods (Li et al., 2014a, 2014b). Because the S-FEM always provide "softer" models than the standard FEM owing to the smoothing techniques, this makes the S-FEM models can achieve more accurate numerical solutions than the FEM models.

In recent years, the S-FEM have been introduced to solve the acoustic problems and coupled structural-acoustic problems (He et al., 2010b, 2011a, 2011b, 2012; Li et al., 2014a, 2014b) and it is found that S-FEM is more effective to control the numerical error than the standard FEM. The present paper is inspired by the work Li et al. (2015) on hybrid smoothed finite element method (HS-FEM) for acoustic problems. This work mainly focused on interior acoustic problems or coupling of structural acoustic problems. As an extension of this, the aim of the current research is to address the problem of scattering of acoustic wave from infinite cylinders in water. In fact, there is a great deal of relevant research can be found in the literatures. Initially, researchers tried to solve the problem mentioned above using analytical method and the geometry of the scatter was also restricted to circular cylinders. Actually, the exact solutions are available only for the scatter with simple geometry. When it comes to more complex and more realistic scatter, one have to resort to numerical methods. Pillai et al. (1982) utilized the *T*-matrix method to handle the sound scattering by rigid and elastic infinite elliptical cylinder in water. DiPerna and Stanton (1994) derived a conformal mapping approach for solving sound scattering by cylinders of non-circular cross section. Recently, Mitri (2015a, 2015b, 2016) used the partial-wave series expansion (PWSE) method to calculate the acoustic scattering and radiation force on a rigid elliptical cylinder in different waves. In addition, some other numerical techniques such as the mode matching method (MMM) (Ikuno and Yasuura, 1978), the Fourier matching method (FMM) (DiPerna and Stanton, 1994), the boundary element method (BEM) (Seybert et al., 1988) and the finite element method (FEM) (Strouboulis et al., 2008) have also been developed to tackle this issue with varying degree of success.

In this paper, the HS-FEM was combined with the DtN boundary condition to give a HS-FEM-DtN model for sound scattering by infinitely long rigid cylinders in water. Both circular and noncircular cross sections are considered in current research. Due to the good performance of the HS-FEM in interior acoustic problems, it is expected that the HS-FEM will solve the exterior acoustic problems with very exact solutions.

This paper is organized as follows: Section 2 introduces the finite element formulation for exterior acoustic problems in unbounded domains. Section 3 briefly describes the Dirichlet to Neumann boundary condition for finite element schemes. Section 4 contains the detailed formulation of the hybrid smoothed finite element method. Dispersion error analysis of acoustic problems using HS-FEM is presented in Section 5. Section 6 outlines the numerical error for acoustic problems. In Section 7, several numerical examples are studied in details. Final conclusions from the numerical results are drawn in Section 8.

2. The finite element formulation for exterior acoustic problems in unbounded domains

As shown in Fig. 1, an exterior acoustic problem in an infinite domain R bounded internally by the surface Γ of an obstacle is considered. We assume that the boundary Γ can be decomposed into two portions Dirichlet boundary condition Γ_p and Neumann boundary condition Γ_v, where $\Gamma = \Gamma_p \cup \Gamma_v$ and $\Gamma_p \cap \Gamma_v = \varnothing$.

The exterior boundary-value problem can be described in the following equations.

$$\Delta p + k^2 p + f = 0 \qquad \text{in } R \tag{1}$$

$$p = g \qquad \text{on} \quad \Gamma_g \tag{2}$$

$$\nabla u \cdot n = h \qquad \text{on} \quad \Gamma_h \tag{3}$$

here p denotes the spatial part of the acoustic pressure or velocity potential, Δ and k represent the Laplace operator and wave number, respectively, f, g and h are given functions.

For exterior acoustic problem in unbounded domains, the Sommerfeld radiation condition which requires only outgoing waves be proportional to $exp(ikr)$ at infinity. The radiation condition requires that energy flux be positive at infinity, this property guarantees that the boundary-value problem has unique solution.

The Sommerfeld radiation condition can be describe as follows:

$$\lim_{r \to \infty} r^{\frac{(d-1)}{2}} \left(\frac{\partial p}{\partial r} - ikp \right) = 0 \tag{4}$$

where d is the spatial dimension.

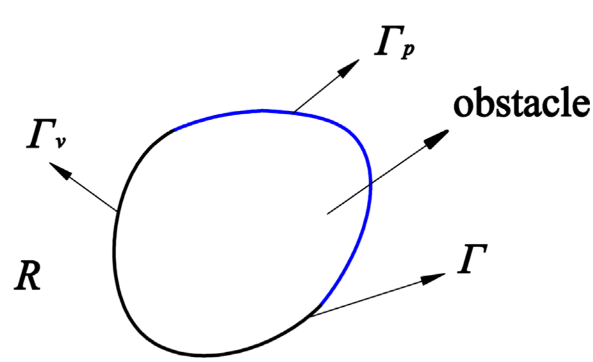

Fig. 1. The exterior acoustic problem in an infinite domain.