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Probability distribution of acoustic scattering from slightly rough sea surface

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ABSTRACT

This paper focuses on statistical characterization of acoustic scattering from slightly rough sea surface with small Rayleigh parameter. Infinite random rough sea surface is irradiated with a plane wave single-frequency source and the field scattered by this surface is calculated by the Method of Small Perturbation (MSP). Then, Probability Density Function (PDF) of the scattered wave is calculated. The calculated PDF is compared with some commonly used distribution. For real cases, due to directivity of the sound source, only a certain part of the rough sea surface is insonified. The PDF of the scattered wave is also found in this case. To find the PDF of the scattered pressure amplitude it is necessary to solve numerical integrals. When the distance between the reference point and the insonified surface is much greater than the wavelength of the incident wave, obtaining a numerical solution to these integrals is impractical. To avoid this problem, we propose in this paper, to calculate the integrals approximately by using the method of stationary phase. In all cases, the calculated distribution for amplitude of the scattered wave is compared with the distribution proposed in the literature.

1. Introduction

One of the most important problems in underwater communications is the lack of an appropriate model describing the underwater channel. This problem is due to the fact that the medium for propagation is too complex and most of its parameters vary with time and space. The surface waves that cause the scattering of acoustic waves incident on the sea surface are the most prominent factors leading to the complexity of the underwater channel (Stojanovic and Preisig, 2009).

Extensive research has been conducted on the scattering of acoustic waves from rough surfaces, based on the laws of wave propagation physics. There are numerous articles addressing this subject. For example, the books (Ogilvy, 1991; Voronovich, 1994) address this subject in detail.

In addition to the aforementioned books, many papers have investigated the statistical characteristics of waves scattered from the sea surface and floor. For instance, Thorsos (1990), Dahl (1998), Mousavi et al. (2012a,b), Karjadi et al. (2012), Heitsenrether and Badiey (2004), and Baktash et al. (2015) treat the mean value and angle of arrival of sea-surface-scattered wave. A summary regarding wave scattering from the sea surface is available in Fortuin (1970).

In some papers, the PDF of the sea-surface-scattered wave is investigated using experiments under specific circumstances, leading to various results depending on the experiment condition. For example, in Chitre (2007) and Radosevic et al. (2009), it is shown by performing experiments in various locations, that the distribution of amplitude fading for different paths is Rayleigh and Rician, respectively. In Yang and Yang (2006), using experiments performed in a different location, it is found that amplitude fading is *K*-distributed.

None of the references (Ogilvy, 1991; Voronovich, 1994; Thorsos, 1990; Dahl, 1998; Mousavi et al., 2012a,b; Karjadi et al., 2012; Heitsenrether and Badiey, 2004; Fortuin, 1970; Chitre, 2007; Radosevic et al., 2009; Yang and Yang, 2006) have presented a closed form expression for the PDF of a sea-surface-scattered wave amplitude.

A few works exist which try to theoretically obtain the distribution of amplitude fading in the presence of several propagation paths. For example, in Geng and Zielinski (1995) it is assumed that different paths from transmitter to receiver consist of a dominant stable component besides many smaller randomly scattered components resulting from the inhomogeneity of the ocean medium. Other scattered components are also considered which are due to the incidence of waves on the sea surface and the sea floor. Thus, using the results obtained in wireless communications, and assuming that many scattered paths exist, it is shown in Geng and Zielinski (1995) that amplitude fading of the channel, when none of the dominant stable components arrive at the receiver and the channel is fully scattering, is Rayleigh-distributed. When both randomly scattered and dominant stable component reach the receiver,

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amplitude fading has a Ricean distribution (Geng and Zielinski, 1995). A reasoning similar to that of Geng and Zielinski (1995) is available in Pollak (1958) and D'Antonio and Hill (1965), where it is shown that the fading statistics is Rayleigh. Albeit, none of the parameters we are faced with in practical situations is discussed in any of Geng and Zielinski (1995), Pollak (1958) or D'Antonio and Hill (1965). Among these parameters are the sea state and the frequency and the incidence angle of the acoustic wave.

In this paper, the PDF of a wave scattered from the sea surface is obtained in closed form. Assuming that the sea surface slightly deviates from a certain mean and the slopes of the waves are sufficiently small, the Method of Small Perturbation (MSP) is used to obtain the field of the scattered wave, and the Pierson–Moskowitz spectrum is used to model the sea surface. As the first step, PDF of surface-scattered waves resulting from a monochromatic plane wave incident on the infinite sea surface is obtained. Then, assuming a directive transmitter, PDF of the scattered wave is formulated when the insonified surface is bounded. To determine the PDF of scattered wave from finite rough sea surface, it is essential to solve numerical integrals. These integrals become more time-consuming as the distance between the reference point and the insonified surface becomes longer. In cases where this distance is much greater than the incident wave wavelength, there is no practical numerical solution to these integrals. In this paper, we calculate these integrals using the method of stationary phase approximately. Then, we utilize these approximations to determine the PDF of the scattered field.

In both cases, in order to depict the PDF, a large number of samples is taken from the acquired distribution and the envelope amplitude distribution is found using the histogram of the scattered field amplitude. In addition, these examples show that these distributions depend on various parameters such as receiver position (reference point) and the relative direction of sea wave propagation.

Note that, in this paper, we have only considered the scattering of acoustic waves incident on the sea surface and scattering of acoustic waves due to inhomogeneity of the ocean medium is neglected. At low frequencies, scattering due to inhomogeneities such as gas bubbles and other turbulences are negligible (Brekhovskikh and Lysanov, 2003; Yang and Yang, 2006). Hence, the results obtained in this paper give the amplitude distribution of scattered field at low frequencies, at least in the case that the scattering due to water inhomogeneities is negligible.

2. Scattering of sound at slightly rough sea surface

Assume that a single-frequency plane wave with amplitude Q and horizontal and vertical components of acoustic wave vector (ξ_{0x}, ξ_{0y}) and γ_0, is incident on the sea surface, with a small Rayleigh parameter expressed as:

$$P = 2k\sigma \cos\theta_0 \tag{1}$$

where k is the wave number of the incident wave, σ is the root-mean-square displacement of the rough sea surface from its mean level, and θ_0 is the angle between the negative z-axis and the incident wave direction. The positive z-axis points to the sea floor and the x–y plane is the sea surface. To obtain an expression for the scattered field of the acoustic wave, we assume that the total sound field in the half-space $z > 0$ is equal to:

$$p(x,y,z) = p_0(x,y,z) + p_s(x,y,z) \tag{2}$$

where p_0 is the acoustic field in the absence of surface roughness, and p_s is the first-order scattered field. Using the boundary condition at the pressure-release surface, the field scattered from the incident plane wave with unit amplitude, for a small Rayleigh parameter is given by Brekhovskikh and Lysanov (2003):

$$\begin{aligned} p_s(x,y,z) = {} & 2iQ\gamma_0 \exp(i(x\xi_{0x} + y\xi_{0y})) \\ & \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [A(\nu_x,\nu_y)\exp[i(x\nu_x + y\nu_y \\ & + z\sqrt{k^2 - (\xi_{0x}+\nu_x)^2 - (\xi_{0y}+\nu_y)^2})]]\mathrm{d}\nu_x \mathrm{d}\nu_y \end{aligned} \tag{3}$$

where $A(\nu_x,\nu_y)$ is the Fourier transform of the sea surface elevation, and

$$\xi_{0x} = k \sin\theta_0 \cos\varphi_0 \tag{4}$$

$$\xi_{0y} = k \sin\theta_0 \sin\varphi_0. \tag{5}$$

In the above equations, φ_0 is the azimuth angle of the acoustic wave incident on the sea surface with respect to x-axis.

Since the sea surface elevation is random, the field scattered by it has also a random value. In order to derive an appropriate distribution function for p_s, a realistic model of the sea surface must be considered. For this purpose, we use the model proposed in Holthuijsen (2007) which exploits the wave-spectrum. In this model the real sea surface is represented as the sum of a large number of harmonic wave components moving in various directions with different amplitudes, phases and periods. Thus, the sea surface elevation $\eta(x,y,t)$ can be written as Holthuijsen (2007):

$$\eta(x,y,t) = \sum_{n=1}^{N}\sum_{m=1}^{M}\{a_{n,m} \times \cos(\omega_n t - k_n x \cos\phi_m - k_n y \sin\phi_m + \alpha_{n,m})\} \tag{6}$$

where for each harmonic component, $k_n = 2\pi/L_n$ is the wave number, L_n is the wavelength, ϕ_m denotes the direction of propagation, and $a_{n,m}$ and $\alpha_{n,m}$ are the random amplitude and the phase, respectively, at each frequency ω_n and direction ϕ_m. Note that all $2NM$ parameters $a_{n,m}$ and $\alpha_{n,m}$ are independent of each other. Frequency and wave number are related in the following way (Holthuijsen, 2007):

$$\omega_n^2 = gk_n \tanh(k_n d) \tag{7}$$

where d is the water depth and g is the gravitational acceleration.

In this model, for each frequency ω_n and direction ϕ_m, the phase $\alpha_{n,m}$ is uniformly distributed over the interval $(-\pi, \pi)$, and the amplitude $a_{n,m}$ has a Rayleigh distribution with parameter $\mu_{n,m}$ varying with frequency and direction:

$$pr(\alpha_{n,m}) = \frac{1}{2\pi} \quad \text{for } -\pi < \alpha_{n,m} \le \pi \tag{8}$$

$$pr(a_{n,m}) = \frac{\pi}{2}\frac{a_{n,m}}{\mu_{n,m}^2}\exp\left(-\frac{\pi a_{n,m}^2}{4\mu_{n,m}^2}\right) \quad \text{for } a_{n,m} \ge 0 \tag{9}$$

where $pr(\cdot)$ denotes the probability density function and

$$\mu_{n,m} = E\{a_{n,m}\}. \tag{10}$$

Note that we have:

$$var(a_{n,m}) = \left(\frac{4}{\pi} - 1\right)\mu_{n,m}^2 = \left(1 - \frac{\pi}{4}\right)E\left\{a_{n,m}^2\right\} \tag{11}$$

The parameter $\mu_{n,m}$ depends on parameters such as wind speed and sea location. Each sea surface model gives a different relation between $\mu_{n,m}$ and these parameters. Examples of these sea surface models are Pierson–Moskowitz (Pierson and Moskowitz, 1964), JONSWAP (Hasselmann et al., 1973) and Bretschneider (Bretschneider, 1959). Some other models are discussed in Holthuijsen (2007) and Reeve et al. (2004). To determine $\mu_{n,m}$ for each wave component, two-dimensional spectrum of wind-generated waves is used, showing how the variance of $\eta(x,y,t)$ is distributed over frequencies and directions. In other words, the two-dimensional spectrum determines the variance of each harmonic wave component. If a finite number of harmonic components is used to