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# Catastrophe characteristics and control of pitching supercavitating vehicles at fixed depths

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ABSTRACT

The existence of the cavity and of a nonlinear planing force results in catastrophe phenomena and causes an instant loss of stability for the supercavitating vehicle during the travel process. To analyze the catastrophe characteristics, the least-squares method is exploited in order to approximate the nonlinear planing force, and the simplified model of the pitching supercavitating vehicle at fixed depths is established. According to the splitting lemma, the cusp catastrophe potential function is built for the pitching supercavitating vehicle at fixed depths and the critical condition of the catastrophe is obtained. A dynamic washout filter-aided feedback is used to control the catastrophe phenomena of the vehicle, and the simulation result shows that the control method is effective and feasible.


## 1. Introduction

A supercavitating vehicle is a type of underwater vehicle that travels through liquid at high speed: the liquid surrounding the body gasifies and forms a supercavity that contains the body, which results in a dramatic drag reduction. Only the cavitator, the fins and the tail of the body have contact with the liquid. Therefore, the dynamic characteristics and the stability of the body are linked to the travel speed and the cavitation state. The nonlinear dynamics of the vehicle, such as bifurcations and chaos with the cavitation number changing, have been addressed in Bai et al. (2008), and Lin et al. (2007, 2005). In actual travel, the nonlinear phenomena will cause transient instability, broken structures, etc., which are the main challenges to overcome when conducting a dynamic analysis and control design for a supercaviating vehicle.

The dynamic analysis and the control design of the supercavitating vehicle have been studied in a series of papers. Dzielski and Kurdila (2003) presented the pitch-plane dynamics of the supercavitating vehicle by assuming a fixed cavitation number, where the oscillatory behavior of the vertical speed and pitch rate are observed in the uncontrolled system. Kirschner et al. (2002) developed a full-state model of the vehicle in which the numerically generated fin force data were used and a forward-feedback scheme was applied to control the vehicle based on a linearized plant. Kulkarni and Pratap (2000) neglected gravity and modeled the planing force as the force generated due to an impact of the aft body with a rigid barrier and a variable coefficient of restitution, also under a fixed cavitation number. A linear control scheme and a switching control scheme were proposed by Lin et al. (2004, 2008) to stabilize the vehicle at a desired equilibrium point (the travel speed and the cavitation number are both constant). Lv et al. (2010) designed a continuous sliding mode controller for the supercavitating vehicle with mismatched uncertainties included, and used an adaptive technology to estimate the unknown upper bound of the mismatched uncertainties. A linear model of the supercavitating vehicle was obtained by the technology of the feed forward control in Fan et al. (2010), where the changing location of the mass center and the uncertainty of the hydrodynamic coefficients and of the cavity deformation was considered. Several nonlinear control approaches, such as the sliding mode control and the quasi-linear-parameter-varying control, were investigated by Mao and Qian (2009) for the supercavitating vehicle of the dive-plane dynamics, and a saturation compensator was designed to compensate for the physical limits of the deflection angles of the control surfaces of the cavitator and fins. The accelerated motion of the supercavitating vehicle was presented in Wang et al. (2010), and a gain schedule controller was designed to stabilize the dive-plane dynamics during the accelerated motion. Unlike a fully wetted vehicle, the supercavitating vehicle involves complicated cavity dynamics, strong nonlinear forces and the nonlinear phenomena as the cavitation number and the travel velocity change. Thus far, most existing dynamic analyses and control designs for supercavitating vehicles focus on a particular choice of system parameters such as the cavitation number and the vehicle

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forward speed, as studied in Dzielski and Kurdila (2003), Kirschner et al. (2002), Kulkarni and Pratap (2000), Lin et al. (2004, 2008), Lv et al. (2010) and Fan et al. (2010). There is a limited amount of published work on the nonlinear behavior of the supercavitating vehicle. Bifurcations with respect to a quasi-static variation of the cavitation number were studied by Lin et al. (2007, 2005), where the system exhibits rich and complex nonlinear dynamics including nonsmooth bifurcations and smooth bifurcations as well as periodic behaviors. A fitting function of the planing force was presented by Zhao et al. (2013) to analyze the catastrophe characteristics. The splitting lemma was utilized to transform a disturbance Hamilton system into a normal Hamilton system, and the fold catastrophe model of the supercavitating vehicle was established.

In this paper, we investigate the catastrophe theory to analyze the catastrophe phenomena of the supercavitating vehicle when it instantly loses stability. In particular, we established a cusp catastrophe model for the pitching supercavitating vehicle at fixed depths. The critical condition of the catastrophe model is analyzed by solving the cusp catastrophe model. In order to solve the issue of the catastrophe phenomena in the system, a dynamic washout filter-aided feedback controller is designed to control the catastrophe.

This paper is organized as follows. A simplified pitch model of the supercavitating vehicle at fixed depths is presented in Section 2. Based on the simplified model, the cusp catastrophe model is established and the catastrophe characteristics analysis is discussed in Section 3. In Section 4, the dynamic washout filter-aided feedback controller is designed to control the catastrophe and the control results are discussed. Concluding remarks are presented in Section 5.

## 2. Mathematical model of a supercavitating vehicle

Consider the pitch-plane dynamics of the supercavitating vehicle. There are four kinds of forces acting on the body: the cavitator lift $F_{cav}$ acting on the cavitator, the gravity $F_g$ of the body acting on the gravity center, the fin lift $F_{fin}$ acting on the fins, and the planing force $F_p$ acting on the aft body (caused by the immersion of the parts at the tail of the body). Variable definitions and coordinate directions are shown in Fig. 1. The reference coordinate is placed at the center of gravity with the positive $x$-axis pointing in the forward horizontal direction and the $z$-axis pointing to the center of the Earth. $\delta_c$ and $\delta_e$ are the cavitator deflection angle and the fin deflection angle respectively, both with respect to the body centerline (body symmetry line). The force components as a function of vehicle states are expressed as follows.

$$\begin{cases} F_{cav} = \frac{1}{2}\pi\rho R_n^2 V^2 C_{x0}(1+\sigma)\left(\frac{w}{V}+\delta_c\right) \\ F_{fin} = -\frac{1}{2}\pi\rho R_n^2 V^2 C_{x0}(1+\sigma)n\left(\frac{w}{V}+\frac{qL}{V}+\delta_e\right) \\ F_g = \frac{7}{9}\rho m\pi R^2 Lg \end{cases} \tag{1a}$$

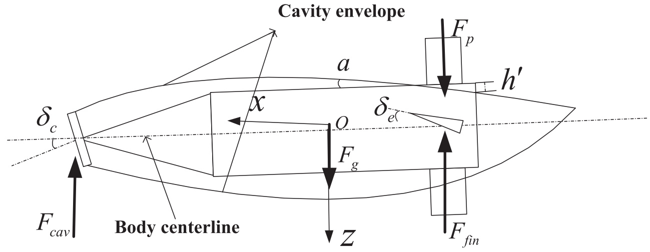


**Fig. 1.** Variables in the dive-plane.

where $\rho$ is the density of water and $m$ is the density ratio of water and the vehicle. $L$ denotes the length of the vehicle, $g$ is the acceleration of gravity and $R_n$ is the radius of the cavitator. $V$ denotes the velocity of the vehicle while $C_{x0}$ is the lift coefficient. $w$ denotes the vertical velocity of the vehicle, $n$ is fin effectiveness, and $\sigma$ is the cavitation number. Hence, the vehicle mass ($M$), and the moment of inertia around the $y$-axis ($I_{yy}$) are given as

$$M = \frac{7}{9}(m\rho\pi)R^2 L \tag{1b}$$

$$I_{yy} = \frac{11}{60}R^4 L\pi\rho m + \frac{1933}{45360}R^2 L^3 \pi\rho m \tag{1c}$$

The force and moment equations around the gravity center using the conventions shown in Fig. 1 are written as

$$M\dot{w} = F_{cav} + F_{fin} + F_g + F_p \tag{1d}$$

$$I_{yy}\dot{q} = -(F_{cav}L_c + F_{fin}L_f + F_p L_f) \tag{1e}$$

where $L_c$ and $L_f$ are the respective moment arms of the forces on the cavitator and the fins, and $F_p$ is the planning force. Using these equations, supplemented by the basic kinematic equations for position and pitch angle, the dynamic equation of the supercavitating vehicle can be obtained as in Bálint et al. (2007) and Dzielski and Kurdila (2003). Here, following the work of Dzielski and Kurdila (2003) and Lin et al. (2005), a four-state model is chosen to study the catastrophe behavior of the system.

$$\begin{pmatrix} \dot{z} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & 1 & -V & 0 \\ 0 & a_{22} & 0 & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & 0 & a_{44} \end{pmatrix} \begin{pmatrix} z \\ w \\ \theta \\ q \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{pmatrix} \begin{pmatrix} \delta_e \\ \delta_c \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ d_2 \\ 0 \\ d_4 \end{pmatrix} \left( -V^2 \left[ 1 - \left( \frac{R_c - R}{h'R + R_c - R} \right)^2 \right] \frac{1+h'}{1+2h'}\alpha \right) \tag{2}$$

where $z$, $w$, $q$, and $\theta$ are the four states of the vehicle. They are the depth at which the body is located, the vertical speed of the vehicle, the pitch rate, and the pitch angle respectively. $R_c$ is the radius of the cavity at the planning location and $R$ is the radius of the vehicle. $h'$ is the immersion depth and $\alpha$ is the immersion angle. The forces $F_{cav}$, $F_{fin}$ and $F_g$ are included in the equation, and due to the combination calculation, required to get the state variants $z$, $w$, $q$, and $\theta$, we get the final expression of Eq. (2). The parameters of expression of $F_{cav}$, $F_{fin}$ and $F_g$ are contained in the coefficients of the matrix. The last portions of the equation contain the planning force and the moment.

The dynamic model of the supercavitating vehicle has strong nonlinear characteristics, mainly caused by the planing force, which is itself caused by the bounce of the vehicle in the cavity. The planing force is

$$F_p = -V^2 \left[ 1 - \left( \frac{R_c - R}{h'R + R_c - R} \right)^2 \right] \frac{1+h'}{1+2h'}\alpha \tag{3}$$

From the expression (3), we can see that the planning force is related to the immersion depth $h'$ of the body's tail, to the immersion angle $\alpha$ and to the feature sizes $R$ and $R_c$. The immersion depth $h'$ and the immersion angle $\alpha$ are both a function of the vertical speed of the body, therefore, the planning force can only be simplified by the nonlinear function of the vertical speed, as seen in Lin et al. (2007). Here, the least-squares method, which finds the fitting function for which the sum of the squares of the errors between the fitting points and the given points is minimal,