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Nonlinearity of abnormal waves by the Hilbert–Huang Transform method

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ABSTRACT

The nonlinearity of full-scale abnormal wave data is studied by the Hilbert–Huang Transform method. The vertical asymmetry of abnormal waves is examined in relation to the level of the intra-wave frequency modulation of different Intrinsic Mode Functions. The Draupner New Year wave data time series is used as an example of a vertically asymmetric abnormal wave, while record 37 from the hurricane Camille dataset is chosen as an example of a vertically symmetric abnormal wave. A measure of intra-wave frequency modulation is proposed based on the instantaneous frequency variations within one single wave. The study of the variation of local frequency in the vicinity of abnormal waves demonstrates that a larger intra-wave frequency modulation is associated with a higher asymmetry of the wave profile.

1. Introduction

Abnormal waves, which are also called freak, giant, or rogue waves, are usually very steep waves with a height much larger than the surrounding waves. The term abnormal wave is given to a wave, which is larger than the ones that would normally be expected by a certain wave theory. Dean (1990) defined abnormal waves as those whose height is outside the expected range of the linear theory. The characteristics of such a wave will depend on the duration of the sea state and on the level of probability beyond which one considers that the occurrence is an outlier of the parent distribution. Based on the 20 min duration of wave record, it was proposed that a wave should be classified as an abnormal if its height is greater than twice the significant wave height. Various suggestions have been made about the criteria for identifying abnormal waves (e.g. Clauss, 2000; Guedes Soares et al., 2003; Kharif et al., 2009).

The shape of the large waves has been a subject of several studies since the impact of large waves on marine structures is of main concern for their safe and reliable operations (e.g. Myrhaug and Kjeldsen, 1986; Guedes Soares et al., 2004a). Wolfram et al. (2001) has found that the abnormal waves are steeper than the significant wave steepness, while Christou and Ewans (2011) have concluded that there is no strict relation between steepness and the abnormal wave definition. Veltcheva and Soares (2012) found that when abnormal waves appear in a group of high waves, they are more symmetrical than when a single abnormal wave occurs.

The asymmetrical profile of abnormal waves observed in real sea conditions suggests that they are an essentially nonlinear phenomenon. Some deterministic and statistical approaches to the study of sea waves consider the asymmetrical waveform as being the result of the presence of harmonics with appropriate phases. This view of nonlinearity is called the Fourier view (Huang et al., 1999). It results from using a linear method (Fourier decomposition) to fit essentially nonlinear data. The harmonic components with fixed amplitudes and frequencies are mathematical ones, and they do not have an analog in the real world. Although the Fourier method is sufficient to obtain the integral characteristics of sea states, it provides only partial information since it suppresses the details of the local time variations of the wave process. The study of the local properties of real sea waves requires methods in which frequency is a function of time or in other words, an instantaneous frequency is needed.

The concept of instantaneous envelope or instantaneous energy of sea waves is well accepted for the statistical analysis of sea waves (e.g. Tayfun, 1983; Hudspeth and Medina, 1988). At the same time, the concept of instantaneous frequency has limited using. Some of the difficulties in accepting the instantaneous frequency are coming from trying to relate the instantaneous frequency to the Fourier frequency. The value of the instantaneous frequency at a given moment of time shows what the frequency at that particular moment of time is, while the Fourier spectrum is about all frequencies, with their corresponding amplitudes, which coexist independent of time. In addition, it is mistakenly assumed

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that a single-valued instantaneous frequency exists at any given instant but a complex signal consists of many different frequencies at any given time.

Instantaneous frequency is defined by the Short Time Fourier Transform (STFT) as a frequency which still varies globally but assumed to be constant within each of the sub-integral spans (Guedes Soares and Cherneva, 2005). This method has a limitation of the Fourier frequencies, which have to be constant in the sub-intervals. The Wigner-Ville distribution defines instantaneous frequency through the mean moment of different components at a given time. The averaging operation, however, results in losing the details required to describe complex multi-component data.

A direct method for defining instantaneous frequency is as a derivative of the phase of the analytical function corresponding to the data time series. It was adopted earlier for the investigation of the local properties of sea waves and wave group structure (e.g. Bitner-Gregersen and Gran, 1983; Huang et al., 1992; Cherneva and Veltcheva, 1993; Cherneva and Guedes Soares, 2001). The instantaneous frequency, however, is physically meaningful only if the data are mono-component. Therefore, in order to determine the instantaneous frequency of an arbitrary signal it is necessary first to decompose the signal into a series of mono-component contributions. This is the purpose of the Empirical Mode Decomposition (EMD). The EMD is a key part of the Hilbert Huang Transform (HHT) method, introduced by Huang et al. (1998). The EMD identifies the specific local time scales and extracts the energy associated with them into a set of Intrinsic Mode Functions (IMFs). A time interval between successive extrema in the time series is defined as a time scale.

This work aims to study the nonlinearity of full-scale abnormal wave data by the HHT method. The interpretation of nonlinearity as an intra-wave frequency modulation was introduced by Huang et al. (1998). The variation of the instantaneous frequency within one single wave is called intra-wave frequency modulation. Here the dependence of the vertical asymmetry of abnormal waves on the intra-wave frequency modulation of different IMFs is examined.

Two wave records containing abnormal waves are studied in detail: the Draupner New Year wave data record as an example of a vertically asymmetric abnormal wave and record 37 from hurricane Camille containing symmetrical abnormal wave. Additionally, the dependence of the intra-wave frequency modulation on the nonlinearity of the abnormal wave is examined using six wave records from the Camille dataset and six wave records from the North Sea storm dataset.

The paper is organized as follows: The method of analysis is presented in Section 2. The intra-wave frequency modulation is discussed using a numerical example. The data are briefly described in Section 3, followed by a presentation of the results of the analysis of the wave records, containing abnormal waves.

2. Hilbert–Huang Transform method

The Hilbert–Huang Transform method consists of two steps: Empirical Mode Decomposition (EMD) and Hilbert spectral analysis. The time series of the sea surface elevation $\eta(t)$ is first decomposed by the EMD into a finite number n Intrinsic Mode Functions $C_j(t), j=1,\dots n$, which extract the energy associated with various intrinsic time scales, and a residual r_n. The superposition of the IMFs and the residue reconstructs the data record:

$$\eta(t)=\sum_{j=1}^{n} C_j(t)+r_n \tag{1}$$

In the second step of the HHT analysis, the Hilbert transform is applied to the IMF:

$$\hat{C}_j(t)=\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{C_j(t')}{t-t'}dt' \tag{2}$$

where P indicates the Cauchy principal value. The amplitudes a_j, phases φ_j and instantaneous frequencies ω_j of the IMF C_j are calculated by

$$a_j(t)=\sqrt{C_j^2(t)+\hat{C}_j^2(t)} \tag{3}$$

$$\varphi_j(t)=\mathrm{arctg}\left(\frac{\hat{C}_j(t)}{C_j(t)}\right) \tag{4}$$

$$\omega_j(t)=\frac{d\varphi_j(t)}{dt} \tag{5}$$

Using (3), (4) and (5) the original data $\eta(t)$ can be expressed as the real part (Re) of the complex expansion

$$\eta(t)=Re\sum_{j=1}^{n} a_j(t)e^{i\int\omega_j(t)dt} \tag{6}$$

where the amplitude and the frequency are functions of time, in contrast to the Fourier decomposition, and they provide at a given moment t the best local fit of the sea surface elevation $\eta(t)$.

The time–frequency distribution of the squared amplitude is designated as the Hilbert energy spectrum $H(t,\omega)$. For simplicity the Hilbert energy spectrum is denoted as the Hilbert spectrum. The Hilbert spectrum allows determining which frequency exists at a particular time, while the Fourier frequency spectrum provides information about which frequency exists generally in a given data series.

There are some additional limitations of using the Hilbert transform for the direct estimation of instantaneous frequency by (5), which are discussed in detail in Huang et al. (2009). The IMF has to be not only mono-component, but also the amplitude modulated (AM) and frequency modulated (FM) parts must be well separated so that their spectra are not overlapping. These limitations are resolved by the normalized Hilbert Transform, proposed by Huang et al. (2009), which aims to split the amplitude modulated and frequency modulated parts of each IMF. The instantaneous frequency can then be correctly determined from the normalized IMF. The normalized Hilbert transform is used here to determine the instantaneous frequency of the Intrinsic Mode Functions.

The instantaneous frequency offers a different view of the nonlinear data. The definition of instantaneous frequency (5) appears to be local, for it is defined by differentiation rather than integration, and hence, the resulting instantaneous frequency describes the time variations of the frequency. The variation of the local frequency within one single wave is denoted as an intra-wave frequency modulation by Huang et al. (1998). If the frequency changes within a wave, the wave profile cannot be described by a simple trigonometric function. Thus if the instantaneous frequency is used, then the nonlinearity is viewed as an intra-wave frequency modulation. This is the Hilbert view of nonlinearity.

Instantaneous frequency was defined in the context of the frequency modulation theory in communications. Modulation is the process of changing the carrier's amplitude, frequency, or phase with a modulating signal that typically contains information to be transmitted. The frequency modulation (FM) is defined as variations of the frequency ω_c of a “carrier” signal with a modulating signal $g(t)$

$$\omega(t)=\omega_c\left[1+\delta g(t)\right] \tag{7}$$