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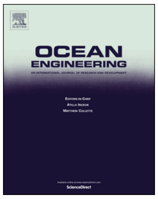

Multi-directional random wave interaction with an array of cylinders

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ABSTRACT

Based on the linear theory of wave interaction with an array of circular bottom-mounted vertical cylinders, systematic calculations are made to investigate the effects of the wave directionality on wave loads in short-crested seas. The multi-directional waves are specified using a discrete form of the Mitsuyasu-type spreading function. The time series of multi-directional wave loads, including both the wave run-up and wave force, can be simulated. The effect of wave directionality on the wave run-up and wave loading on the cylinders is investigated. For multi-directional waves, as the distribution of wave spreading becomes wider, the wave run-up at some points around the cylinders is found to increase. This suggests that multi-directional wave run-up tends to be larger than unidirectional wave run-up. In addition, the wave directionality has a significant influence on the transverse force. The biggest transverse force is found to occur on the rear cylinder rather than the front one. This is quite different from the results in unidirectional waves and should be paid much attention in the design of offshore structures. At last, the possibility of the near-trapping under the multi-directional random waves is investigated. It is found that the near-trapping also occurs for multi-directional wave conditions.

1. Introduction

Circular cylinders are often used in offshore engineering and many offshore structures are comprised of arrays of cylinders. Examples include bridges, wind turbine foundations, offshore platforms and floating airports. In ocean engineering design, wave loading is an important factor. There are two regimes for calculating the wave loads on a cylinder, depending on the slenderness parameters D/λ, with D is the diameter of a cylinder and λ is the wave length. If $D/\lambda > 0.15$, wave diffraction is important and should be considered, otherwise it can be ignored.

For wave loads on a large cylinder, a superposition eigenfunction expansion method was used by MacCamy and Fuchs (1954) to obtain a linear solution, based on the assumption that the incident wave has a small steepness. For the case of waves acting upon an array of cylinders, the effect of a given cylinder on the incident wave will produce a scattered wave which will in turn be scattered by adjacent cylinders. Thus the computation of the velocity potential must account for the diffraction of the incident wave field by each body and the multiple scattering from other bodies. An exact solution for the diffraction of linear water waves by arrays of bottom-mounted, vertical circular cylinders was first given by Spring and Monkmeyer (1974) using a direct matrix method. It represented an extension of the single cylinder case presented by MacCamy and Fuchs (1954). Further, an approximate solution to this problem was given by McIver and Evans (1984) in which they assumed that the cylinders were widely spaced. An accurate algebraic method was developed by Kagemoto and Yue (1986) to calculate the hydrodynamic properties of a system of multiple three-dimensional bodies in water waves. Subsequently, a simplified expression for the velocity potential in the vicinity of a particular cylinder was developed by Linton and Evans (1990) which led to simple formulae for the first-order and mean second-order wave forces on multiple cylinders as well as the free surface profile.

Based on wave tank experiment and numerical methods, a number of researchers have studied wave interaction with an array of cylinders. Ohl et al. (2001a, 2001b) studied regular and irregular wave interaction with an array of cylinders, and a very good agreement between the theory and the laboratory results was found. Ma et al. (2001a, 2001b) studied fully nonlinear wave diffraction around a pair of fixed cylinders in a numerical wave tank based on the finite element method (FEM). A semi-analytical solution was developed by Huang (2004) for second-order wave diffraction by an array of cylinders in monochromatic waves. It was found that at relatively high frequencies, the enhancement of the second-order component in the wave run-up was much more general than on the forces. Wang and Wu (2010) developed a fully nonlinear numerical wave tank to simulate three-dimensional waves and wave–structure interactions by the finite element

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Nomenclature

a	cylinder radius
A	wave amplitude for calculated waves
A_0	incident wave amplitude
d	water depth
D	cylinder diameter
f_x	normal wave force
f_y	transverse wave force
F_x	non-dimensional significant normal wave force
F_y	non-dimensional significant transverse wave force
$G(f, \theta)$	directional spreading function
$H_{1/3}$	significant wave height
i	imaginary unit, $i^2=-1$
k	wave number
L	distance between two adjacent cylinders
m_0	zeroth moment of wave spectrum
N	number of cylinders
k	wave number
R	non-dimensional wave run-up
s	directional spreading parameter
$S(f)$	wave frequency spectrum
$S(f, \theta)$	wave directional spectrum
$T_{1/3}$	significant wave period
T_p	peak wave period
α	angle around cylinder
β	angle for the incident wave
γ	peak enhancement factor of JONSWAP spectrum
λ	wave length
σ_θ	standard deviation of directional spreading
ω	wave angular frequency

method. The effect of the tank wall on waves and forces was investigated, and the nonlinear features of waves and forces were also discussed. Govaere et al. (1999, 2001) derived linear wave transformation due to the presence of an impermeable cylindrical pile protected by a series of submerged permeable structures and wave loads on the pile were studied. Zhao et al. (2010) investigated the diffraction of waves by an array of porous circular cylinders based on the linear wave theory and model test.

On the other hand, some researchers focused on the phenomenon of near trapping. Maniar and Newman (1997) considered a long array of cylinders (up to 101 cylinders). They found that when the wave number was close to the nearly trapped mode, a very large hydrodynamic force could arise on the cylinders in the middle of the array. Evans and Porter (1997, 1999) found that for the wave forces on circular arrays of four, five and six cylinders, the near trapping phenomenon also existed. Malenića et al. (1999) further showed that similar behavior could occur for the second-order result. Duclos and Clément (2004) extended this work to consider arrays of unevenly spaced cylinders, displaced randomly from a regular array according to a disordering parameter. They focused on two effects of this spacing irregularity, reduction of peak forces associated with the trapped mode phenomena, and regularization of the transmission coefficient for waves propagating through the arrays. Kagemoto et al. (2013, 2014) studied the second-order resonance among an array of two rows of cylinders by experiment and theoretical calculation. This work found that large free-surface displacements could be induced for special wave conditions.

However, most of the research associated with wave interaction with arrays of cylinders has been focused on unidirectional waves. But in reality, sea waves are multi-directional waves. In multi-directional sea condition, the wave directionality could lead to quite different wave–structure interaction results compared with the unidirectional wave fields. Yu et al. (1996) investigated the wave force due to multi-directional random waves on a small vertical cylinder by experiment. The variation of various hydrodynamic coefficients with KC number and wave directional spreading was investigated. Lee et al. (2007) used a numerical model to predict the interaction of multi-directional random surface waves with rectangular submarine pits. Liu et al. (2010, 2012) solved the modified Boussinesq equations based on a finite element model with unstructured triangular elements, and considered the effects of wave directionality on the wave run-up on a group of cylinders. Ji et al. (2013, 2015a, 2015b) systematically investigated the multi-directional random wave forces and run-up on a large cylinder by experimental and numerical methods. It was found that a small directional spreading parameter (high directionality), gives rise to a large transverse force which should not be ignored. The dynamic response of a mini-Tension Leg Platform under multi-directional wave conditions was studied by Niedzwecki et al. (2001). Li et al. (2012, 2014) studied the interaction of multi-directional focused waves with a vertical cylinder by experiment. The effect of a multi-directional focused wave on wave loads was investigated. The interaction of waves with porous circular cylinder and cylinders were studied theoretically by Govaere and Silva (2002) and Silva et al. (2003), and then it was extended to unidirectional and multi-directional waves, it was found that as the spectrum broadens in frequency and angle, the modulation around the structure damps faster.

In the present paper, the superposition method is used to study the multi-directional random wave loads on an array of large-scale bottom-mounted vertical cylinders. Linton and Evans' method is used as the transfer function, which is combined with multi-directional random waves to predict the interactions with arrays of cylinders. Considering that there are rather few references in the literature about real sea wave loads on an array of cylinders, our main concern in this paper is focused on the effect of the wave directionality on the multi-directional wave run-up and the force loading on a cylinder array. The goal is that these results will provide a reference to improve the design of offshore structures.

2. The model of multi-directional random wave loads on cylinders

2.1. Wave interaction with cylinders in regular wave conditions

Under the assumption of linear wave theory, the incident wave velocity potential can be described as

$$\Phi(x,y,z,t)=\mathrm{Re}\left\{-\frac{igA}{\omega}\frac{\cosh(z+d)}{\cosh kd}\phi_{inc}(x,y)e^{-i\omega t}\right\} \tag{1}$$

where A represents the amplitude of the incident wave, d the water depth, and the wave number k and the wave frequency ω should satisfy the dispersion relation

$$\omega^2=gk\tanh kd \tag{2}$$

in which g is the gravitational acceleration.

A sketch of the situation is shown in Fig. 1, which shows an array of bottom-mounted vertical cylinders with an incident wave with an angle β. We assume that there are N ($N\geq 1$) fixed vertical circular cylinders, so $N+1$ coordinate systems will be used: (r, θ) are polar coordinates in the (x, y)-plane centered at the global