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Development and application of a semi-analytical method with diagonal coefficient matrices for analysis of wave diffraction around vertical cylinders of arbitrary cross-sections

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ABSTRACT

This paper proposes a semi-analytical method for modeling short-crested wave diffraction around a vertical cylinder of arbitrary cross-section, in an unbounded domain. In this method, only the boundaries of domain are discretized using special sub-parametric elements. The formulation of elements is constructed by employing higher-order Chebyshev mapping functions and special shape functions. The shape functions are introduced to satisfy Kronecker Delta property for the potential function and its derivative, corresponding to the governing Helmholtz equation of the problem. Furthermore, the first derivative of shape functions of any given control point are set to zero. By implementing weighted residual method and using Clenshaw–Curtis numerical integration, the coefficient matrices of equations system become diagonal, yielding a set of decoupled governing Bessel differential equations for the whole system. In other words, the governing equation for each degree of freedom (DOF) is independent of other DOFs of the domain. Accuracy and efficiency of present method are fully demonstrated through three short-crested wave diffraction problems which are successfully modeled using a few numbers of DOFs (or nodes), with excellent agreements between the results of the present method and those of other analytical/numerical solutions.

1. Introduction

One of the most important subjects in the design of offshore structures is the calculation of the wave loads on vertical cylinders, as offshore platforms are mostly constructed with vertical cylinders. The wave forces on vertical cylinders can be calculated using the well-known Morison's equation, if the wavelength of incident waves is much greater than the cross-sectional dimensions of cylinders. The interaction of wave and vertical cylinders has been widely investigated analytically, numerically, and experimentally. For the first time, an analytical solution for the diffraction of plane waves around a cylinder with circular cross-section was presented by MacCamy and Fuchs (1954). The accuracy of the results was then verified with some experimental tests by Chakrabarti and Tam (1975) and Neelamani et al. (1989). Goda and Yoshimura (1972) presented an analytic solution for the diffraction of plane waves around an elliptical cylinder by solving the Helmholtz equation using the method of separation of variables.

The finite element method (FEM) was the first numerical method used to solve wave diffraction problems (Bettess and Zienkiewicz, 1977). Modeling of unbounded domain for the infinite radiation boundary condition (BC) is one of the main challenges in the FEM analyses. To overcome this problem, a hybrid method became very popular in the literature (see for example, Houston (1981), Tsay and Liu (1983), and Tsay et al. (1989)). The hybrid method combines the advantages of FEM solution in bounded domain and those of analytical solution in unbounded BC.

The boundary element method (BEM) requires basically reduced surface discretizations, and is appealing alternative to the FEM for solving wave diffraction problems (see for example, Au and Brebbia (1983), and Lesnic et al. (1993)). The BEM models unbounded domain very well. However, the BEM requires a fundamental solution to the governing differential equation to obtain the boundary integral equation. In other words, the BEM requires fundamental solutions that are dependent on the problem of interest. Although the coefficient matrices of the BEM are much smaller than those of the FEM, they are usually non-symmetric, non-positive definite and fully populated.

The scaled boundary finite element method (SBFEM) was successfully developed to solve wave diffraction problems, by combining the advantages of the FEM and the BEM (Li et al., 2006; Tao et al., 2007; Liu et al., 2012; Meng and Zou, 2012, 2013).

A semi-analytical method for solving potential (Khaji and Khodakarami, 2011) and elastostatic (Khodakarami and Khaji, 2011; Khaji

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and Khodakarami, 2012) problems has been recently developed. In this paper, the mentioned method is developed for the Helmholtz equation to solve the problem of wave diffraction by a vertical cylinder in an unbounded domain. Compared to previous works of the authors, it should be noted that the physical nature of the Helmholtz equation is completely different from the Laplace equation that is applied in potential problems. In other words, the Helmholtz equation is intrinsically devoted to dynamic phenomena, while the Laplace equation principally deals with static/semi-static problems such as steady state heat conduction ones. Furthermore, elastostatic problems are corresponding to solid mechanics; but, the present manuscript utilizes the proposed method in ocean engineering to model the interaction problem of water waves with vertical cylinders. In this method, only the boundaries of the unbounded domain are discretized. Furthermore, Bessel differential equations are derived from governing equation of motion for the unbounded domain. In this method, higher-order Chebyshev mapping functions as well as special shape functions are used. By employing weighted residual method and Clenshaw–Curtis numerical integration, coefficient matrices of equations' system become diagonal. Compared to the BEM, the BEM needs fundamental solutions in order to evaluate the required coefficient matrices and obtain the solutions. The fundamental solutions are dependent to the considered problems and vary from problem to problem. Moreover, the coefficient matrices of the BEM are usually fully populated. On the other hand, there are neither fundamental solutions nor populated matrices in the proposed method. Consequently, the mentioned advantages of the present method as well as higher order polynomials used as mapping and shape functions may result in less computational cost for the present method in comparison with the BEM. In summary, the computational cost for solving problems using decoupled governing equations and exactly diagonal coefficient matrices may be less than classical numerical method such as the FEM, the BEM, and the SBFEM, which are based on lower-order elements and populated matrices. The accuracy and efficiency of the present method are then demonstrated using three numerical examples.

2. Problem definition

A monochromatic wave train traveling in the $+x$ direction is considered on a fixed vertical cylinder extends from the sea bed to the above water free surface of the ocean along the z axis (see Fig. 1). The origin is located at the center of the cylinder on the mean water surface. Employing the assumption of the linearized wave theory, the total velocity potential Φ satisfies Laplace equation as

$$\nabla^2\Phi = 0 \quad \text{in} \quad \Omega, \tag{1}$$

where Ω is the problem domain. The total velocity potential Φ is equal to the summation of the velocity potential of the incident wave Φ^I, and the velocity potential of scattered wave Φ^s (i.e., $\Phi = \Phi^I + \Phi^s$). The linearized combined free surface BC and the sea bed condition may be given by the following well-known relations:

$$\Phi_{,tt} + g\Phi_{,z} = 0 \quad \text{at} \ z = 0, \tag{2}$$

$$\Phi_{,z} = 0 \quad \text{at} \ z = -h, \tag{3}$$

in which g denotes the gravitational acceleration. In addition, the subscript comma indicates the partial derivative with respect to the variable following the comma.

The velocity potential may be decomposed by separating the variable z and the time variable t from the remaining variables as given by the following expressions:

$$\Phi(x,y,z,t) = \phi(x,y)Z(z)\exp(-i\omega t) \tag{4}$$

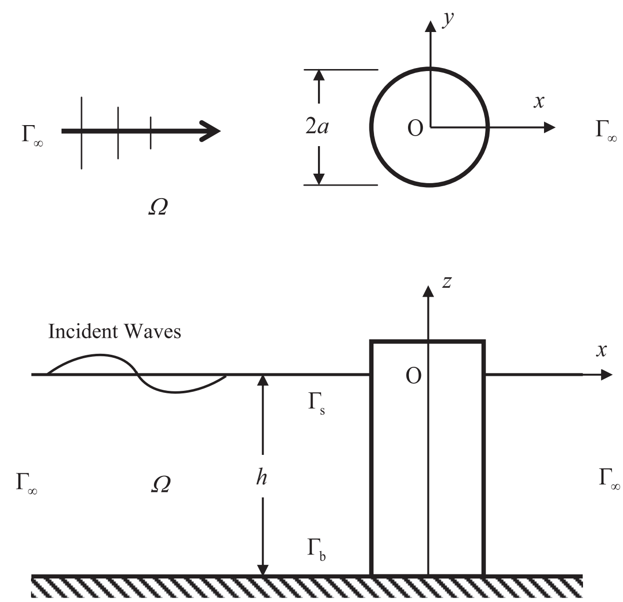

Fig. 1. Plan view (top) and elevation view (bottom) of the problem of wave diffraction around a vertical circular cylinder.

$$\Phi^I(x,y,z,t) = \phi^I(x,y)Z(z)\exp(-i\omega t) \tag{5}$$

$$\Phi^s(x,y,z,t) = \phi^s(x,y)Z(z)\exp(-i\omega t) \tag{6}$$

where ω is the wave angular frequency, $i = \sqrt{-1}$, and $Z(z)$ implies the corresponding water depth function as given below

$$Z(z) = \frac{\cosh k(z+h)}{\cosh kh}, \tag{7}$$

where k and h are the wave number and the water depth, respectively.

Using the separation of variables technique, the sea bed condition can be satisfied automatically. Furthermore, the linear free surface BC is fulfilled by implementing the following dispersion relationship

$$\omega^2 = gk\tanh(kh). \tag{8}$$

Therefore, the problem becomes two-dimensional (2D) at the free surface. The relation between the total velocity potential, the incident wave velocity potential, and the scattered wave velocity potential may be easily derived as:

$$\phi(x,y) = \phi^I(x,y) + \phi^s(x,y). \tag{9}$$

For the diffraction partition of the problem, the velocity potential $\phi^s(x,y)$ is governed by the well-known Helmholtz equation as:

$$\nabla^2\phi^s + k^2\phi^s = 0. \tag{10}$$

In addition, the Neumann BC and so-called Sommerfeld condition are respectively applied at the interface of fluid and structure, and the radiation condition at infinity as:

$$\phi^s_{,n} = -\phi^I_{,n} \quad \text{on} \quad r = a, \tag{11}$$

$$\lim_{kr\to\infty}\sqrt{kr}\left(\phi^s_{,r} - ik\phi^s\right) = 0, \tag{12}$$

where r is the radial axis, and n denotes the unit normal to the boundary.

For short-crested incident waves traveling in the x direction, the velocity potential can be obtained from the real part of following relation (Fuchs, 1952):

$$\phi^I = -\frac{igA}{\omega}Z(z)\exp(i(k_x x - \omega t))\cos(k_y y), \tag{13}$$