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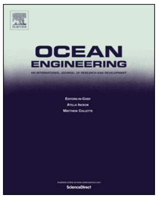

Numerical investigation on the dynamics of two bubbles

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ABSTRACT

Two-bubble interaction has wide applications, such as cavitation, seabed exploration and underwater explosions, in which bubble collapse and jetting are the most focused issues. Based on the incompressible potential flow theory coupled with the boundary integral method, we investigate the dynamics of two differently sized bubbles. After jets penetrate the two bubbles, vortex rings model is introduced to simulate the subsequent evolution of two toroidal bubbles. To validate the present model, the results are compared with the experiment recorded by a high-speed camera and the data in a published literature. Favorable agreements of bubble shape history are observed. Then, the influence of the size ratio l(defined as $l=R_{m,1}/R_{m,2}$, $R_{m,1} \le R_{m,2}$; R_m is the maximum radius attained in a free field) on bubble behaviors is investigated numerically. Annular jet leading to bubble splitting, axial jet formation towards and away from the larger bubble are observed. It is also found that the minimum jet velocity of the smaller bubble and the strongest jet impact of the larger bubble are obtained when l is around 0.8. Finally, the two-bubble interaction is classified in a graph according to the size ratio and the inter-bubble distance. The splitting phenomenon occurs only when $l \le 0.5$.

1. Introduction

Bubble dynamics has been one of the active fields for many years due to its importance in various physical and engineering problems, including airgun bubbles for seabed exploration (Cox et al., 2004), cavitation bubbles on the blades of propellers and turbomachines (Blake and Gibson, 1987), and underwater exploration bubbles (Cole, 1948; Liu et al., 2014). The violent bubble collapse plays an important role in the above mentioned fields, and jetting is always believed to be one of primary causes for surface damage and erosion in most cases. Interest in bubble collapse pattern and jet formation is also stimulated by the observations in many works that, bubble splitting occurs under certain conditions (Chahine, 1982; Blake et al., 1986; Brujan et al., 2001). However, bubbles do not appear in isolation in most situations. The existence of a bubble provides a special boundary condition to another one, and dynamics of the bubble collapse, no doubt, will change. The understanding of the interactions of multiple bubbles is fundamental and significant.

The key experimental studies on two-bubble interaction are those of Lauterborn (1982), Tomita et al. (1990), Bremond et al. (2006), Fong et al. (2009) and Chew et al. (2011). Using a pulsed laser system, Lauterborn (1982) pointed out that the repulsion between two bubbles produced with a time delay led to jets away from each other and then several vortex rings were generated with an especially stable one. Tomita et al. (1990) demonstrated the bubble–bubble interaction was significantly influenced not only by the relative size but also the mutual distance between them. They observed an earlier jet of the small bubble entered the large bubble due to the inertia of water in the experiments (called 'jet coalescence' in this paper). Bremond et al. (2006) triggered the expansion of nuclei on a solid surface by an impulsive lowering of the liquid pressure, and coalescence and jet towards were observed by employing the setup to study the dynamics of two and more bubbles in a row. Fong et al. (2009) studied the complex interaction between two similarly sized bubbles (the differences kept below 15%) and broadly classified the interactions in a graph according to the relative inter-bubble distance and the phase difference. In the experiments, four types of behavior of bubble–bubble interaction were observed, namely, jet towards, jet away, coalescence and a behavior termed the 'catapult effect'. The study was extended by Chew et al. (2011) to the interaction of differently sized bubbles using high speed photography. The above four types of bubble behavior were observed in the experiments, and a demarcation curve was drawn in graph of phase difference against size difference to predict the direction of water jet. They found that smaller bubble behaved similarly to a single bubble oscillating near a free surface with large size difference. Besides, the responses of two bubbles to nearby boundaries were also studied

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experimentally (Blake et al., 1993; Lal and Menon, 1996; Cho et al., 2000; Robinson et al., 2001; Chew et al., 2013).

With the development of computer science, bubble dynamics has been widely studied using numerical methods, among which the boundary integral method (BIM) is the most popular due to its efficiency and high precision (Blake et al., 1986; Wang et al., 1996; Best, 1993; Cox et al., 2004; Zhang et al., 2013a). Bubbles evolve to toroidal geometries when the jet impacts on the opposite surface of the bubble. To allow for the transition from a singly connected to a doubly connected fluid domain after jet impact, the further motion of a toroidal bubble is simulated by introducing a vortex ring model (Wang et al., 1996) or by inserting a domain-cut across the liquid jet (Best, 1993). In multiple bubbles interactions, complex phenomena were observed in earlier studies. The complexity renders the numerical study difficult. In previous works, therefore, the computations using the axisymmetric BIM were always limited to the bubble evolution before jet impact (Blake et al., 1993; Robinson et al., 2001; Pearson et al., 2004a). In the numerical calculations by Fong et al. (2009), they further simulated the bubble motion after jet impact by ignoring the toroidal bubble. Obviously, computational results will be more accurate with the motion of the toroidal bubbles considered.

In multiple bubbles interactions, bubble dynamics after jet impact involves the interaction between a toroidal bubble and a singly connected one, even toroidal bubbles interaction. Nevertheless, few computational studies are concerned with the above interactions. Therefore, the first objective of this paper is to further simulate the toroidal phase following the liquid jet impact to give insight into bubble behaviors.

Obviously, size difference between two bubbles (different oscillation time) leads to more complex phenomena. If two non-buoyancy bubbles are in the same size, each bubble behaves like a single one near a plane rigid wall placed midway between the two bubbles (Tomita et al., 1990). For bubbles with large size difference, the smaller bubble behaves similarly to a single bubble oscillating near a free surface (Chew et al., 2011). We note the change in jet direction with the increase of size difference. Therefore, the second objective of this paper is to investigate how size difference affects bubble collapse pattern and jet formation.

The present work is therefore intended to perform a parametric numerical study of the influence of the size ratio on bubble behavior regimes, where some other phenomena not covered previously will be investigated. After jet impact, the further evolution of a toroidal bubble is traced through the implementation of a vortex ring model. In Section 2, the mathematical model and initial conditions of two bubbles are outlined. Next, the present model is validated by comparing the simulated results with both the experimental observation recorded by a high-speed camera and the data in a published literature in Section 3. With a fixed inter-bubble distance, a range of bubble behaviors depending on the size ratio is observed in Section 4 while the calculations provide information on the velocity and pressure fields, aiming to reveal much of the underlying physics. In Section 5, much effort is directed towards discussions about bubble collapse and jet formation. Besides, classification of phenomena is presented in a graph according to size ratio and inter-bubble distance. Finally, conclusions are summarized.

2. Mathematical formulation

2.1. Boundary integral method

In this paper, we present a numerical study of the interaction of two differently sized bubbles in an axisymmetric configuration. Because of the short oscillation period (in the order of milliseconds) and the large Reynolds number (Blake et al., 1986; Cox et al., 2004), viscous forces are negligible. And the compressibility is also ignored in the first cycle of oscillation though the compressible effect accounts for the damped oscillation of a bubble (Wang, 2014). The fluid is assumed to be invisid, incompressible and irrotational and thus the velocity potential φ satisfies the boundary integral equation:

$$\zeta\varphi(\boldsymbol{p}) = \iint_S \left(\frac{\partial \varphi(\boldsymbol{p}))}{\partial n} \frac{1}{r_{pq}} - \varphi(\boldsymbol{q}) \frac{\partial}{\partial n}\left(\frac{1}{r_{pq}}\right)\right) dS \tag{1}$$

where the surfaces of two bubbles are denoted by S, $\boldsymbol{p}$ and $\boldsymbol{q}$ are the fixed point and the source point located on the liquid–gas interfaces, ζ with a value of 2π is the solid angle at $\boldsymbol{p}$, the normal derivative is defined as $\partial/\partial n = \boldsymbol{n} \cdot \nabla$ and $\boldsymbol{n}$ is the unit outward normal.

Surface tension is not considered in this paper and the internal pressure of bubble is only related to its initial state which is assumed to obey the ideal gas law. According to the pressure equilibrium on the bubble surface, the flow field pressure just outside the bubble surface P_f can be written as a function of the bubble volume, yielding (Best, 1993; Wang et al., 1996):

$$P_f = P_c + P_{0,i}\left(\frac{V_{0,i}}{V_i}\right)^{\iota} \quad (i = 1, 2) \tag{2}$$

where P_c is the constant vapor pressure with $P_c = 2338Pa$, $P_{0,i}$ and $V_{0,i}$ are the initial pressure and volume of the bubble, respectively, where the subscript i denotes bubble number, ι is the ratio of the specific heats of the gas which is taken as $\iota = 1.25$ for underwater explosion or spark-generated bubble and as $\iota = 1.4$ for air bubble (ideal gas).

Dynamic boundary condition on the bubble surface can be written (Wang and Khoo, 2004):

$$\frac{d\varphi}{dt} = \frac{|\nabla\varphi|^2}{2} + \frac{P_\infty}{\rho} - \frac{P_f}{\rho} - g r_z \tag{3}$$

where P_∞ is the ambient pressure on the plane of the bubble center at inception, we choose $P_\infty = P_{atm} = 1.01 \times 10^5$ Pa in this paper, ρ is the fluid density, g is the gravitational acceleration, and r_z is the vertical component of the position vector $\boldsymbol{r}$. $R_{m,i}(i = 1, 2)$ is defined as the maximum radius bubble attained in a free field, and we choose $R_m = \max(R_{m,i}), i = 1, 2$ as our length scale and P_{atm} as pressure scale. Unless stated, dimensionless quantities are used in the following context. Thus the dimensionless form of Eq. (3) is written as:

$$\frac{d\varphi}{dt} = \frac{|\nabla\varphi|^2}{2} + 1 - P_c^* - \sigma_i\left(\frac{V_{0,i}}{V_i}\right)^k - \chi^2 r_z \tag{4}$$

where dimensionless parameters are defined as: $P_c^* = P_c/P_{atm}$, the strength parameter $\sigma_i = P_{0,i}/P_{atm}$ and the buoyancy parameter $\chi = \sqrt{\rho g R_m / P_{atm}}$. In the current paper, we consider the motion of millimeter sized bubbles and hence set $\chi = 0$. Besides, the inter-bubble distance is defined as $d_{in} = D_{in}/R_m$ where D_{in} is the distance between the bubble centers at inception of a vertical column of two bubbles.

The kinematic boundary condition on all surfaces is (Wang et al., 1996):

$$\frac{d\boldsymbol{r}}{dt} = \nabla\varphi \tag{5}$$

2.2. Modeling for toroidal bubbles

In the two-bubble interaction, bubble would transit from a singly-connected to a doubly-connected form after jet impact. The idealized impact model (Best, 1993) is used to dispose the nodes of