Ocean Engineering 106 (2015) 341–353

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Bivariate distributions of significant wave height and mean wave period of combined sea states

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ARTICLE INFO

Article history:
Received 19 November 2014
Accepted 8 July 2015

Keywords:
Joint probability distribution
Significant wave height
Mean period
Marginal distribution

ABSTRACT

This work presents the results of the fit of three bivariate models to twelve years of significant wave height and mean zero-crossing period data of swell, wind sea components, and combined sea states from Australia. The Conditional Modelling Approach defines the joint distribution from a marginal distribution of significant wave height and a set of distributions of mean zero-crossing period conditional on significant wave height. The second model fits the Plackett model to the data, and the last one applies the Box–Cox transformations to the data with the aim of making it approximately normal to fit a bivariate normal distribution to the transformed data. The conditional model with a lognormal distribution for the significant wave height and lognormal distributions for the zero-crossing period gave the best fit for the total sea states and for the wind component. In case of the swell component the conditional model with a Weibull distribution to the significant wave height and a lognormal distribution to the mean zero-crossing period gave a relatively close fit to the data.

1. Introduction

The joint distribution of the spectral parameters that govern the sea states, namely the significant wave height (H_s) and the mean (T_z) or peak period (T_p), is an essential element to be able to derive the design wave loads for marine structures.

Several methods have been adopted to study bivariate distributions describing the wave climate (e.g. Repko et al., 2004). One of the first approaches was proposed by Ochi (1978), who adopted the bivariate lognormal distribution, resulting from an exponential transformation of the bivariate Normal distribution. This approach although elegant and simple to apply, requires that the logarithm of the data looks normally distributed, and although this may happen for low and moderate H_s it starts not being applicable for large wave heights. A bivariate lognormal with correction for skewness (Fang and Hogben, 1982) was an attempt to improve the bivariate lognormal model. A measure of skewness was included in a term modifying the lognormal form of the marginal distribution of H_s.

A model based on the marginal distribution of H_s and conditional distributions of T_z (or T_p) is an intuitive one that increases the flexibility of modelling and has been adopted by Haver (1985), Guedes Soares et al. (1988), and Bitner-Gregersen and Haver (1989).

The marginal distribution of H_s is the most important one that governs the intensity of the loads induced on marine structures and it is thus the starting point of this approach. Several approaches have been adopted to model the marginal distribution as summarized for example by Isaacson and Mackenzie (1981), Muir and El-Shaarawi (1986) or Guedes Soares and Scotto (2001). Then the various marginal distributions of T_z conditional on H_s must be fit to the data so as to build the bivariate distribution.

Haver (1985) used a combination of lognormal distribution for lower values of H_s and Weibull for the tail region, calling it Lonowe distribution of H_s, and adopted lognormal distributions for T_z (or T_p), while Mathisen and Bitner-Gregersen (1990) used the 3 parameter Weibull distribution for H_s and lognormal distributions for T_z (or T_p), and concluded that this joint distribution performed better than other joint H_s and T_p distributions.

Athanassoulis, et al. (1994) have proposed the use of the Plackett bivariate model as a systematic and simple way of fitting the bivariate distribution functions of H_s and T_z. The structure of the Plackett model, even though not being completely general, allows the specification of any two marginal distributions and leaves the subsequent modelling of the dependence structure to be made with the estimation of a parameter related to the correlation between the variables.

Prince-Wright (1995) proposed maximum likelihood models of joint environmental data. The main idea of this method is to use a transformation of a joint environmental data set to a Gaussian model using a variant of the transformation of Box and Cox (1964) and to evaluate the transformation parameters by the maximum

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http://dx.doi.org/10.1016/j.oceaneng.2015.07.010

Nomenclature

H_s	significant wave height
T_p	peak period
T_{02}, T_z	mean zero-crossing period
m_0	zero order moment
m_2	second order moment
$m_{0,T}$	total zero order moment of the two components
$m_{0,s}$	zero order moment of the swell component
$m_{0,w}$	zero order moment of the wind sea component
$T_{02,T}$	total mean zero-crossing period
$T_{02,s}$	mean zero-crossing period of the swell component
$T_{02,w}$	mean zero-crossing period of the wind sea component
$H_{s,T}$	total significant wave height
$H_{s,s}$	significant wave height of the swell component
$H_{s,w}$	significant wave height of the wind sea component

likelihood method. Bitner-Gregersen et al. (1998) have compared this approach with the conditional modelling approach proposed by Bitner-Gregersen and Haver (1989), showing that each one has advantages and problems. The main advantage of the conditional modelling approach is the fact that it is more flexible and in several cases one environmental parameter, e.g., significant wave height, dominates the loading which means that the errors in this model are not so crucial. Also an advantage of this model is that for a data set which has a low correlation coefficient, the model enables to increase the weight in the fit of the data range of interest. But the problem is that, there is no theoretical method for choosing the best form of defining the joint function. The main advantage in the case of the Maximum Likelihood Model (MLM) is that by modelling the simultaneous data as multivariate normal random variables, using the Gaussian transformation, a joint density is defined a priori (Bitner-Gregersen et al., 1998). A disadvantage in this model is that it is affected by the adopted procedure for transforming variables from the physical to the normal space, and the model does not allow putting emphasis on one environmental parameter. It was shown in Bitner-Gregersen et al. (1998) that the conditional modelling approach and the joint model based on the Box–Cox transformations have a similar performance with respect to the waves.

Bitner-Gregersen and Guedes Soares (1997) presented an overview of probabilistic modelling approaches, which was later updated by Guedes Soares and Scotto (2011), and by Bitner-Gregersen (2011) who presented an exhaustive review of the joint long term probabilistic modelling of met-ocean parameters giving particular attention to the Conditional Modelling Approach (CMA). The application of these joint environmental models to different datasets presented their accuracy and their limitations. The models, Maximum Likelihood Model (MLM) and the CMA use the complete probabilistic information obtained from observations of the variables, e.g., significant wave height, thus these models, seem to be the most suitable for establishing joint probabilities.

Ferreira and Guedes Soares (2000, 2002), used a kernel density to model transformed data, respectively in the univariate and bivariate cases. This has the advantage of avoiding the choice and estimation of parametric models which will impose a specific behaviour of the data. The data was first transformed by a Box–Cox transformation, which has performed well in transforming long term wave data in regression models (Cunha and Guedes Soares, 1999), even in the case of bivariate models of H_s and T_z (Guedes Soares and Cunha, 2000). Athanassoulis, and Belibassakis (2002) have also proposed the use of kernel densities as being flexible approaches for univariate and multivariate data. Soares and Guedes Soares (2007) have compared the application of these approaches to a specific data set, clarifying thus the differences in their performance.

The approaches just discussed have modeled the sea states as being described by one set of $H_s - T_z$. However, Guedes Soares (1984) have studied combined sea states composed of a swell and a wind sea component, showing that they have a large probability of occurrence (Guedes Soares 1991, Lucas, et al. 2011). As demonstrated by several authors (e.g., Guedes Soares, 1984; Guedes Soares and Nolasco, 1992) in several situations the sea states are a result of the combination of more than one wave system, and the spectrum exhibits in this case two peaks. The double-peaked wave spectra can be observed whenever a swell system joins with a locally wind-driven system. The swell is characterized by low frequency wave systems that were generated in distant storms. This component of the spectrum is represented by the low frequency peak in the measured wave spectra. As demonstrated by Guedes Soares (1991), the percentage of occurrence of a double-peaked spectrum could be of the order of 25% in different occasions, however, the probability of occurrence tends to decrease as the significant wave height of the sea state would increase.

This implies that a proper modelling of the wave climate would need to represent both types of sea conditions as already noted by Guedes Soares and Nolasco (1992) and also by Bitner-Gregersen (2005). In fact, Guedes Soares and Nolasco (1992) have demonstrated that these two sets of data follow different probabilistic models.

The need of having a long term distribution for both components of combined sea states is felt when conducting reliability studies, which need the description of both wave components, as shown by Teixeira and Guedes Soares (2009). Weather vanning floating platforms tend to align to the main wave direction and can be sensitive to other wave systems from other directions which can excite roll motion and affect their operationally, especially in the case of LNG platforms in particular if this occurs during offloading operations (Pessoa et al., 2015; Sun et al., 2015). To determine the conditions for response in the combined sea states it is necessary to have different models for the larger waves to which the platform will align and the lower seas that will excite roll motion from the transverse direction. This was in fact the motivation that started this study.

This work revisits the bivariate modelling of sea states, applying it both to the total sea state as defined by one pair of $H_s - T_z$ and to each separate set of swell and of wind sea components. It has been observed that T_p is a more discriminating period descriptor than T_z and several joint distribution models use it (e.g. Haver, 1985). However in the case of combined sea states there will be 2 or 3 different T_p, one for each wave system, and thus the T_p of the overall sea state becomes a meaningless variable. This is the reason why this work adopts the characteristic period of the sea state T_{02}, obtained from the zeroth and second moment of the spectrum, although designating as mean period T_z, it is calculated often from the spectral description being more accurately designated as the mean zero-crossing period T_{02}.

This paper starts by describing the three methods for the bivariate description, which are then applied to a data set from Australia, with the characteristics that almost all sea states have a swell and a wind sea component. The models are applied to the total significant wave height and total wave period and to the