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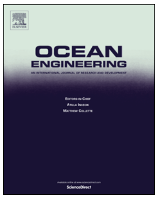

A head loss model for slurry transport in the heterogeneous regime

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ABSTRACT

Although sophisticated 2 and 3 layer models exist for slurry flow (here the flow of sand/gravel water mixtures), the main Dutch and Belgium dredging companies still use modified Durand and Condolios (1952) and Fuhrboter (1961) models, while the main companies in the USA use a modified Wilson et al. (1992) model for heterogeneous transport. These older models use one term for the excess pressure losses, the pressure losses resulting from the solids, with an empirical character. A new model has been developed based on energy considerations. The model consists of two terms for the excess pressure losses, one for the potential energy losses and one for the kinetic energy losses. This gives more flexibility matching experimental results. Although the model is derived fundamentally, the slip velocity of the particles is still an unknown in the model. An equation for this slip velocity is derived, based on physical parameters. The resulting model is validated with numerous experimental data from the literature and from the Delft Dredging Laboratory and matches very well. The advantage of this model is, that it requires the parameters known to the dredging industry and is thus easy to use.

1. Introduction

Although sophisticated 2 and 3 layer models exist for slurry flow (here the flow of sand/gravel water mixtures), the main Dutch and Belgium dredging companies still use modified Durand and Condolios (1952) and Fuhrboter (1961) models, while the main dredging companies in the USA use a modified Wilson et al. (1992) model for heterogeneous transport. When asked why these companies do not use the more sophisticated models, they answer that they require models that match their inputs and they feel that the 2 and 3 layer models are still in an experimental phase, although these models give more insight in the physics. Usually the companies require a model based on the particle size distribution or d_{50}, the pipe diameter D_p, the line speed v_{ls}, the relative submerged density R_{sd} and the temperature (the viscosity of the carrier liquid ν_l). Parameters like the bed associated hydraulic radius are not known in advance and thus not suitable. Usually the dredging companies operate at high line speeds above the limit deposit velocity (LDV) in the heterogeneous or homogeneous regime. This implies that the bed has dissolved and 2 and 3 layer models are not applicable anyway.

Still there is a need for improvement, since the existing models give reasonably good predictions for small diameter pipes, but not for large diameter pipes as used in dredging. Recent projects require line lengths up to 35 km with 5 to 6 booster pumps and large diameter pipes. Choosing the number of booster pumps and the location of the booster pumps depends on the head losses. However it should be considered that the slurry transport process is not stationary. Densities may vary from a water density of 1 t/m^3 to densities of 1.6 t/m^3 and particle size distributions will change over time. This results in a dynamic process where pumps, pump drives and slurry transport interact. The fundamental 2 and 3 layer models require a stationary approach, while the more empirical equations may take the dynamic effects as time and place averaged effects into account. The question is whether a semi empirical approach is possible, covering the whole range of pipe diameters and giving the empirical equations a more physical background, but still using the parameters available to the dredging industry.

The paper first gives a short introduction of the DHLLDV model, followed by the derivation of the slip velocity equation. Simplified models for small, medium and coarse particles are derived for sands and gravels, followed by a comparison with the Durand and Condolios (1952) equation. Finally the application of the model derived for graded sands and gravels is shown.

2. The Delft Head Loss & Limit Deposit Velocity model

The Delft Head Loss & Limit Deposit Velocity (DHLLDV) model is such a model (see Miedema and Ramsdell, 2013, 2014b). The model is based on constant spatial volumetric concentration C_{vs}

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Nomenclature

C_v	Volumetric concentration
C_{vs}	Spatial volumetric concentration
C_{vt}	Transport or delivered volumetric concentration
C_D	Particle drag coefficient
$\sqrt{(C_x)}$	Durand particle Froude number reversed
d	Particle diameter (m)
d_{50}	The 50% cumulative particle diameter (m)
d_m	The mean particle diameter according to Fuhrboter (m)
D_H	Hydraulic diameter of cross-section above bed
D_p	The pipe diameter (m)
E_{rhg}	Relative excess hydraulic gradient
$\Delta E_{s,kin,p}$	Kinetic energy loss particle per collision/interaction (N m)
F_L	Durand critical velocity Froude number
Fr_{DC}	Durand and Condolios Froude number
Fr_p	Particle Froude number, based on the terminal settling velocity (m/s)
g	Gravitational constant (9.81) (m/s^2)
i_l	Hydraulic gradient liquid
i_m	Hydraulic gradient mixture
ΔL	Length of pipe segment (m)
m_p	Mass of particle (kg)
Δp_m	Mixture pressure loss (kPa)
Δp_l	Liquid pressure loss (kPa)
R_{sd}	Relative submerged density
S_{hr}	Relative terminal hindered settling velocity
S_{rs}	Relative slip velocity squared
u_*	Friction velocity (m/s)
v_{ls}	Line speed (m/s)
v_{sl}	Slip velocity (m/s)
v_t	Terminal settling velocity
v_l	Velocity of the liquid/mixture above the bed (m/s)
v_b	Velocity of a sliding bed (m/s)
c	Proportionality coefficient kinetic energy
α_1	Power of ratio thickness viscous sub-layer to particle diameter
α_2	Power of ratio terminal setting velocity to velocity top of viscous sub-layer
α_3	Power of Durand particle Froude number
α_k	Kinetic energy factor
α_m	Momentum based energy factor
α_t	Transition region energy factor
β	Richardson and Zaki hindered settling power
δ_v	Thickness viscous sub-layer m
κ_c	Asymmetrical concentration distribution factor (about 0.5–1)
λ_l	Darcy–Weisbach friction factor liquid-pipe wall
λ_b	Darcy–Weisbach friction factor liquid-bed
ρ_l	Water or liquid density (t/m^3)
ρ_s	Density of the solids (t/m^3)
μ_{sf}	Sliding friction coefficient about 0.4
ν_l	Kinematic viscosity liquid (m^2/s)

curves for uniform sands and gravels and covers 5 main flow regimes, which do not all have to occur in a specific situation

1. A stationary or fixed bed (with or without sheet flow and/or suspension) at very low line speeds.
2. A sliding bed (with or without sheet flow and/or suspension) at low line speeds.
3. Heterogeneous transport a medium (operational) line speeds.
4. Homogeneous transport at high (operational) line speeds.
5. Sliding flow transport at high line speeds and very coarse particles $d > 0.015 \cdot D_p$.

Fig. 1 shows the 5 main flow regimes in an E_{rhg} (relative excess hydraulic gradient) versus i_l (liquid hydraulic gradient) graph. The fixed bed regime may transit to the heterogeneous regime directly for small particles. If $d > 0.15 \cdot D_p$, the sliding flow regime will occur instead of the heterogeneous regime, with a smooth transition. The relative excess hydraulic gradient E_{rhg} and the liquid hydraulic gradient i_l are defined as

$$E_{rhg} = \frac{i_m - i_l}{R_{sd} \cdot C_{vs}} \quad or \quad E_{rhg} = \frac{i_m - i_l}{R_{sd} \cdot C_{vt}} \quad and \quad i_l = \frac{\lambda_l \cdot v_{ls}^2}{2 \cdot g \cdot D_p} \tag{1}$$

The industry usually requires constant transport or delivered concentration C_{vt} curves. By means of a hold up or slip velocity model, constant spatial volumetric concentration curves can be transformed into constant transport volumetric concentration curves, which allows the determination of the bed height in the pipe. The DHLLDV model also contains a sophisticated limit deposit velocity (LDV) model consisting of 5 different sub-models for very fine, fine, medium, coarse and very coarse particles. The LDV is defined here as the line speed above which no stationary or sliding bed is present. The limit of stationary deposit velocity (LSDV) is the transition from a fixed to a sliding bed.

The stationary or fixed bed model, resulting in an explicit equation for the Darcy–Weisbach friction factor λ_b for sheet flow, can be found in Miedema and Ramsdell (2014b). Based on the Wilson (1988) and the Matousek and Krupicka (2010) experiments, complemented with experiments in the Laboratory of Dredging Engineering an empirical explicit equation has been developed for the relation between the Darcy–Weisbach friction factor and the different parameters involved. This equation is

$$\lambda_b = 0.83 \cdot \lambda_l + 0.37 \cdot \left(\frac{(v_l - v_b)}{\sqrt{2 \cdot g \cdot D_H \cdot R_{sd}}} \right)^{2.73} \cdot \left(\frac{\rho_s \cdot \frac{\pi}{6} \cdot d^3}{\rho_l \cdot 1^3} \right)^{0.094}$$

$$= 0.83 \cdot \lambda_l + 0.37 \cdot Fr_{DC}^{\;2.73} \cdot \left(\frac{m_p}{\rho_l} \right)^{0.094} \tag{2}$$

The fixed and sliding bed are analyzed by Miedema and Ramsdell (2014a), resulting in a sliding bed approach different from Wilson et al. (1992). The hydrostatic normal stress approach of Wilson et al. (1992) may be valid up to a bed occupying 50% of the pipe, but for thicker beds this approach is rejected. In the new approach the total normal stress of a bed on the pipe wall varies from 100% of the weight up to about 130% of the weight when the bed occupies 50% of the pipe. For thicker beds this remains about 130%. Now assuming an internal friction angle φ of 30° for loose sand (the angle of natural repose), giving an external friction angle δ of about 20°, a basic friction coefficient μ_{sf} of 0.364 is found. With the factor 1.3 the maximum effective friction coefficient (based on the weight of the bed) is 0.473. Since the process is dynamic and volumetric spatial concentrations are time and place dependent, a weighted average sliding friction factor of about 0.416 is