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A precise computation method of transient free surface Green function

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ABSTRACT

The transient free surface Green function (TFSGF) is of great importance in the prediction of unsteady ship motions with forward speed. In this numerical approach, the wave part of the TFSGF and its spatial derivatives are obtained by solving the fourth-order ordinary differential equations (ODEs) based on the semi-analytical Precise Integration Method (PIM). Theoretical derivations show that in fact the horizontal and vertical derivatives can be expressed by TFSGF, which means only one ODE needs to be solved. The stability and accuracy of this method is demonstrated by the comparison with other method as well as the analytical solutions. Additionally, the proposed method is applied to solve the radiation problem of a floating hemisphere at zero speed using the time domain Rankine–Green method. The numerical hydrodynamic coefficients show good agreement with the analytical solutions.

1. Introduction

The accurate predictions of wave induced motions and hydrodynamic loads are essential in ship design. Large motions or large loads can cause damage to the ship . Therefore development of theories to determine wave induced motions and wave loads has attracted lots of researchers for many decades.

Considering the computational efficiency, the frequency domain and time domain panel method based on potential theory are widely adopted when the viscosity is ignored. Since the free surface Green function itself satisfies the linear free surface condition, it is usually taken as the integral kernel, and then only the wetted body surface needs to be integrated.

Compared to the frequency domain consideration for forward speed problem (Guevel and Bougis, 1982), the time domain approach is found suitable, because the transient free surface Green function (TFSGF) is relatively easy to compute (Datta et al., 2011). Cummins (1962) first discussed the unsteady ship motions in the time domain, and the idea is still current. To carry out the time domain analysis, the three dimensional TFSGF given by Wehausen and Laitone (1960) are widely used. Liapis (1986) discussed the radiation problem of ships with forward speed in the time domain by introducing the impulse response function, and King (1987) added the corresponding wave exciting force in its appropriate convolution form. Lin and Yue (1991) proposed the numerical solutions for large-amplitude ship motions with forward speed in the time domain. Via the close examination of the asymptotics of the numerical solutions in the time domain, Bingham (1994) presented that the linearized problem has a finite solution at the critical frequency corresponding to Brard number $\tau = 1/4$. Besides, Bingham et al. (1994) and Korsmeyer and Bingham (1998), among others, pursued variants of the same method for different classes of 3D forward speed problems.

The key problem for the forward speed time domain simulation is how to compute the wave part of the TFSGF and its derivatives, accurately and efficiently in terms of integral form. Beck and Liapis (1987) divided the computational domain into a number of regions according to the oscillating properties of the integral part, and using the analytical formula as well as the series expansions on three different regions to solve the TFSGF and its derivatives. King (1987) added an additional region where Bessel function expansion was used. Based on the work of Newman (1985), Lin and Yue (1991) developed an improved approach, where ascending series, asymptotic expansion or combination of them and 2-D economized polynomial approximations were used. Huang (1992) proposed a two parameter interpolation technique which was adopted for the calculation of the TFSGF. The efficiency was improved, but the accuracy reduced. Based on the double parameter variables in bounded domain, Clement (1998) uncovered that the TFSGF is the solution of a fourth-order ordinary differential equation (ODE), the same conclusion was derived by Duan and Dai (2001) and Liang et al. (2007). The fourth-order Runge–Kutta method (RK44) was widely adopted to solve the ODEs, but it will lead to stability problems after a long time simulation. Chuang et al. (2007) proposed a semi-analytical method to solve the ODEs, the accuracy and stability were improved, but the required terms is more than forty when the source point and field point are both

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on the free surface. The computation of the TFSGF accurately and efficiently is still a challenge.

The aim of this paper is to develop an accurate and efficient method to evaluate the wave part of the TFSGF and its spatial derivatives, by solving the fourth-order ODE. First, the three ODEs which are used to compute the wave part of TFSGF and its spatial derivatives are reduced to one through theoretical derivation. Then, by modifying the form of the ODE related to TFSGF and its temporal derivatives, the non-homogeneous and stationary system is obtained, which will be solved by the Precise Integration Method (PIM) (Wan-Xie, 2004). In addition, the accuracy and efficiency of the proposed method is verified by comparison with the analytical solutions as well as other method, e.g., RK44. Finally, to further verify the accuracy of the proposed method, it was used to solve the radiation problem of the hemisphere at zero speed, and the numerical solutions of the added mass and damping coefficients show good agreement with the analytical solutions (Hulme, 1982).

2. Mathematical formulation

The linearized assumptions of the free-surface flow problems are made: the fluid is inviscid and incompressible, the flow is irrotational, the pressure is constant over the free surface, and the surface tension is neglected. Then the time domain wave-body interaction problems can be solved by the boundary element method (Hess, 1964), using the TFSGF as the Green function.

Denoting a field point $p(x,y,z,t)$ and a source point $q(\xi,\eta,\zeta,t')$, then the 3D infinite water depth TFSGF given by Wehausen and Laitone (1960) can be written as

$$G(p,q;t-t')=\delta(t-t')\times G_0+H(t-t')\times\tilde{G} \tag{1}$$

with

$$G_0(p,q)=\frac{1}{r}-\frac{1}{r'} \tag{2}$$

and

$$\tilde{G}(p,q,t-t')=2\int_0^\infty\sqrt{gk}\times e^{k(z+\zeta)}J_0(kR)\sin\left[\sqrt{gk}(t-t')\right]\mathbf{d}\,k \tag{3}$$

$$R=\sqrt{(x-\xi)^2+(y-\eta)^2} \tag{4}$$

$$r=\sqrt{R^2+(z-\zeta)^2} \tag{5}$$

$$r'=\sqrt{R^2+(z+\zeta)^2} \tag{6}$$

where δ is the Dirac impulse function, H is the Heaviside step function, and J_0 is the first kind Bessel function of zeroth order.

G_0 is referred as the instantaneous part of the TFSGF, while $\tilde{G}$ is called the wave part. The instantaneous part is easy to evaluate by adopting the Hess Smith method (Hess, 1964). Due to the oscillating properties of the wave part, $\tilde{G}$ is hard to compute. Therefore the main focus of this paper is on the precise computation of the wave part. Clement (1998) proved that the wave part of the TFSGF is the solution of a fourth-order ODE, which will be solved by the Precise Integration Method in this paper. For computation requirement, Eq. (3) is transformed into a non-dimensional form

$$\widehat{G}(\mu,\tau)=\int_0^\infty\sqrt{\lambda}\,\sin\left(\sqrt{\lambda}\tau\right)e^{-\lambda\mu}J_0\left[\lambda\left(\sqrt{1-\mu^2}\right)\right]d\lambda \tag{7}$$

by introducing

$$\lambda=kr' \tag{8}$$

$$\mu=-\frac{z+\zeta}{r'} \tag{9}$$

$$\tau=\sqrt{\frac{g}{r'}}(t-t') \tag{10}$$

Through non-dimensional transformation, the wave part of the TFSGF can be expressed as a function of two real variables μ and τ:

$$\tilde{G}(p,q,t-t')=2\sqrt{\frac{g}{r'^3}}\widehat{G}(\mu,\tau) \tag{11}$$

By a similar method, the horizontal derivative $\tilde{G}_R$ can be written in the non-dimensional form, too.

$$\tilde{G}_R(p,q,t-t')=-2\sqrt{\frac{g}{r'^5}}\widehat{G}_R(\mu,\tau) \tag{12}$$

$$\widehat{G}_R(\mu,\tau)=\int_0^\infty\sqrt{\lambda^3}\,\sin\left(\sqrt{\lambda}\tau\right)e^{-\lambda\mu}J_1\left[\lambda\left(\sqrt{1-\mu^2}\right)\right]d\,\lambda \tag{13}$$

Then the horizontal derivatives of the TFSGF, $\partial\tilde{G}/\partial x$ and $\partial\tilde{G}/\partial y$ can be obtained from

$$\frac{\partial\tilde{G}}{\partial x}=\frac{x-\xi}{R}\tilde{G}_R \tag{14}$$

$$\frac{\partial\tilde{G}}{\partial y}=\frac{y-\eta}{R}\tilde{G}_R \tag{15}$$

The vertical derivative of the TFSGF can also be written in the non-dimensional form:

$$\tilde{G}_Z(p,q,t-t')=2\sqrt{\frac{g}{r'^5}}\widehat{G}_Z(\mu,\tau) \tag{16}$$

$$\widehat{G}_Z(\mu,\tau)=\int_0^\infty\sqrt{\lambda^3}\,\sin\left(\sqrt{\lambda}\tau\right)e^{-\lambda\mu}J_0\left[\lambda\left(\sqrt{1-\mu^2}\right)\right]d\,\lambda \tag{17}$$

and the vertical derivative of the TFSGF can be computed from

$$\frac{\partial\tilde{G}}{\partial z}=\tilde{G}_Z \tag{18}$$

Clement (1998) proved that the form of double parameter function $A_{v,l}(\mu,\beta)$, defined as

$$A_{v,l}(\mu,\tau)=\int_0^\infty\lambda^l e^{-\lambda\mu}J_v\left(\lambda\sqrt{1-\mu^2}\right)\sin\left(\sqrt{\lambda}\tau\right)d\lambda\quad 0\le\mu\le 1 \tag{19}$$

is the solution of the following fourth-order differential equation

$$\frac{\partial^4 A_{v,l}}{\partial\tau^4}+\mu\tau\frac{\partial^3 A_{v,l}}{\partial\tau^3}+\left(\frac{\tau^2}{4}+\mu(3+2l)\right)\frac{\partial^2 A_{v,l}}{\partial\tau^2}+\left(l+\frac{5}{4}\right)\tau\frac{\partial A_{v,l}}{\partial\tau}+\left((l+1)^2-v^2\right)A_{v,l}=0 \tag{20}$$

So the TFSGF and its derivatives can be obtained by solving the three fourth-order ODEs.

From Eq. (7) and Eq. (17), it can be seen that the vertical derivative of the TFSGF can be expressed as a function of $\widehat{G}$,

$$\widehat{G}_Z(\mu,\tau)=-\widehat{G}^{(2)}(\mu,\tau) \tag{21}$$

where $\widehat{G}^{(k)}$ denotes the kth-order differentiation of $\widehat{G}$ with respect to τ.

Introducing two functions: $F_1=J_0\left(\lambda\sqrt{1-\mu^2}\right)e^{-\lambda\mu}$ and $F_2=J_1\left(\lambda\sqrt{1-\mu^2}\right)e^{-\lambda\mu}$. Then the derivative of $\widehat{G}(\mu,\tau)$ with respect to τ gives:

$$\frac{\partial}{\partial\tau}\widehat{G}(\mu,\tau)=-\frac{2}{\tau}\int_0^\infty\sin(\sqrt{\lambda}\tau)\left[\frac{3}{2}\lambda^{\frac{1}{2}}F_1-\lambda^{\frac{3}{2}}\left(\mu F_1+\sqrt{1-\mu^2}F_2\right)\right]d\,\lambda \tag{22}$$