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Using residual areas for geometrically nonlinear structural analysis

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ABSTRACT

Incremental–iterative methods are widely used for tracing the equilibrium paths of structures. To determine the nonlinear structural response, an iteration process is required. In this paper, some residual areas are employed for the base of iteration steps. By setting each area to zero, and minimizing its perimeter separately, some new constraint equations can be achieved. After developing the related formulations, several geometric nonlinear analyses of frames, shell and trusses are performed to evaluate the robustness of the suggested methods. Findings prove the high capability of the authors' first scheme compared to other new proposed methods and cylindrical arc-length technique. Additionally, the capacity of each strategy in passing the limit points is assessed.

1. Introduction

It is well-known that some structures change their initial shapes significantly before reaching their ultimate strengths. In the case of large displacements, the geometric nonlinear analysis is required for assessing accurately the structural behavior. Up to now, various methods have been proposed to reach the nonlinear structural responses. The Newton–Raphson algorithm is a well-known load control approach, which however cannot trace load–displacement curves after a snap-through point (Bergan and Soreide, 1978). To overcome this difficulty, the displacement control tactic was presented by Argyris (1965). It is proved that for structures with displacement limit points, the aforesaid technique converges to wrong responses (Crisfield, 1981). Wempner (1971) and Riks (1972) suggested a more robust scheme named arc-length method. The load increment is constant in each step of the load control approach. On the other hand, the displacement increment remains invariable in the displacement control strategy. Conversely, both the load and displacement increment are updated in each iteration of the arc-length approach. By adding a new unknown parameter to this solution procedure, an extra constraint equation is required.

So far, Investigators have suggested different constraint relationships for various nonlinear equation solvers. In the normal plane method, the locus of the iterative analyses' points is perpendicular to the tangent which passes through the first step equilibrium point (Riks, 1979). Based on the assumptions of Forde and Stiemer (1987), the vectors passing through the previous equilibrium points and the iteration surface points are perpendicular to the location of the iterative points in the updated normal plane approach. One of the prominent ways, which has been frequently utilized for nonlinear analysis of structures, is named cylindrical arc-length method. Crisfield (1981) reformulated Riks' updated tactic by ignoring the force component against the displacement components in the constraint equation. In another well-known strategy, which is called work control method, Yang and McGuire (1985), and Chen and Blandford (1993) assumed that the work increment is constant at the beginning of each incremental step, and is equal to zero in the iterative steps. It is worth emphasizing that one of the important procedures used for tracing the equilibrium path is the generalized displacement control technique. Yang and Shieh (1990), and Richard Liew et al. (1997) showed that this scheme can pass both loads, and displacements limit points.

Other tools have also been used to find the nonlinear response of structures. For instance, the normal flow scheme is based on the iterative analysis performed on the lines normal to Davidenko's flow (Allgower and Georg, 1979). Toklu (2004), and Toklu et al. (2013) optimized total potential energy by using various algorithms. Ritto-Corrêa and Camotim (2008) reviewed the arc-length method and other quadratic control techniques. In a different way, Rezaiee-Pajand and Alamatian (2008), and Rezaiee-Pajand and Sarafrazi (2011) conducted the nonlinear analysis with the help of the dynamic relaxation method. In this approach, the structural static equations are converted to the dynamic equations. Besides, Saffari and Mansouri (2011) and (2012) took advantage of the analytical method to study the nonlinear behavior of structures. It was found that this scheme's order of convergence is four. Recently, Saffari et al. (2013b) combined the Newton–Raphson technique with three other methods, and investigated the geometric and material nonlinear behavior of space trusses.

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Nomenclature

D	Displacement
λ	Load factor
F	internal force vector
R	unbalance load vector
P	External load
K^{n-1}	Tangential stiffness matrix in step $(n-1)$
ΔD_1^n	Displacement increment in first iteration of step n
$\Delta\lambda_1^n$	Load factor increment in first iteration of step n
δD_i^n	Displacement increment in iteration i of step n
$\delta D''^n_i$	Displacement increment caused by unbalance load in iteration i of step n
$\delta D_i' n$	Displacement increment caused by external load in iteration i of step n
$\delta\lambda_i^n$	Load factor increment in first iteration of step n
A_i^n	residual area
s^n	vector passing through point placing on equilibrium path and iteration point
n_i^n	locus of the iterative points
r_{i-1}^n	Reduced residual load in iteration $(i-1)$ of step n
P_i^n	residual perimeter
L_n	Arc length
J_D	Selected number of iteration
J_{n-1}	Number of iterations in step $(n-1)$
A	cross-sectional area
E	elasticity modulus
I	moment of inertia

Two types of nonlinear structural behavior are studied by investigators, which are called material or geometric nonlinearities. In material nonlinearity, the constitutive relation describing the material is nonlinear, and the behavior is affected by physical phenomena such as plasticity. This physical phenomenon requires the satisfaction of yield function in any step of the analysis. Furthermore, the residual loads, which result from material constitutive relation, should be considered in each load step. In geometric nonlinear problems, nonlinearity is due to changes of structural shape, arising from large strains and/or rotations. This behavior affects strain–displacement relationship, and leads to residual loads in any step of analysis. Most of the nonlinear solvers consider the residual values of quantities, such as load, displacement, work, etc., and try to make them vanish in any of the load steps.

Since constraint conditions play a very important role in any nonlinear solvers, this paper is devoted to present three new constraint equations for geometric nonlinear analysis of structures. These novel formulas are obtained by using the residual areas in the iteration process. To reach good convergence criteria, authors will take advantage of the properties of some geometrical shapes. In addition to present the related formulations, the capabilities of the proposed approaches in passing the limit points are evaluated and obtained results are compared with the cylindrical arc-length method.

2. Solving nonlinear equations

Under quite general assumptions, the nonlinear static equilibrium equations of a structure can be put in the following form:

$$R(D,\lambda)=\lambda P-F(D) \quad (1)$$

In this relation, the displacement vector, load factor and external force vector are denoted by D, λ and P, respectively. Also, the internal force and the unbalance load vector are shown by $F(D)$ and $R(D,\lambda)$, correspondingly. The general scheme of incremental-iterative methods is shown in Fig. 1. It is assumed that the $(n-1)$th static equilibrium point $(\Delta D^{n-1},\lambda^{n-1}P)$ is known, and the analysis process is carried out in the (n)th step. The first approximation of the incremental step$(\Delta D_1{}^n,\Delta\lambda_1{}^n P)$, called the predictor step, is obtained by solving the following equation set:

$$K^{n-1}\Delta u_1^n=\Delta\lambda_1^n\, P \quad (2)$$

where $K^{n-1}$is the tangential stiffness matrix at the $(n-1)$th equilibrium point. By deploying this equality, the state of the first point in the (n)th step is calculated. To reach to the (n)th point of the static equilibrium path, sequential iterations are required. In

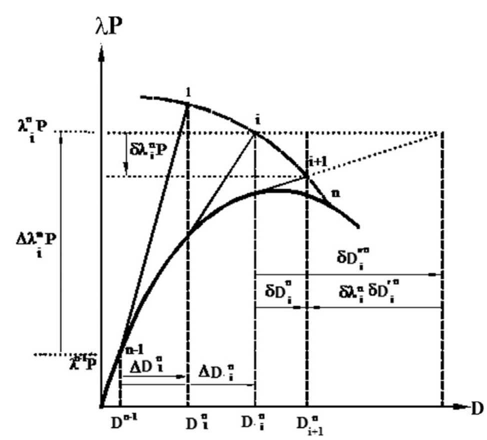

Fig. 1. The general scheme of incremental-iterative methods.

each iteration, the load factor is obtained by using a suitable constraint equation, which defines a surface in the load–displacement space. The correction to the displacement increment in the (i)th iteration is defined as the residual displacement, $\delta D_i{}^n=\Delta D_{i+1}^n-\Delta D_i^n$. Batoz and Dhatt (1979) expressed the residual displacement in the form of a linear combination

$$\delta D_i^n=\delta D''^n_i+\delta\lambda_i^n\delta D_i' n \quad (3)$$

where $\delta D''^n_i$ and $\delta D_i' n$are the displacement induced by residual force and the external load, respectively. The succeeding relationships are correspondingly utilized to calculate these displacements.

$$K_i^n\delta D''^n_i=R_i^n \quad (4)$$

$$K_i^n\delta D_i' n=P \quad (5)$$

The residual force in the (i)th iteration can be obtained by using Eq. (1). It is clear that the residual force, and the external load are known at the beginning of each iteration. Hence, only $\delta\lambda_i^n$ is required to compute δD_i^n. Recall that $\delta\lambda_i^n$ can be obtained by using the constraint equation. Thus, the incremental load and displacement are obtained as:

$$\Delta\lambda_{i+1}^n=\Delta\lambda_i^n+\delta\lambda_i^n \quad (6)$$

$$\Delta D_{i+1}^n=\Delta D_i^n+\delta D_i^n \quad (7)$$

It is worth emphasizing that the constraint equation has to be applied to find the load factor in each step. Due to this fact, the differences of the various incremental–iterative methods are