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A general slip-line field solution for the ultimate bearing capacity of a pipeline on drained soils

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ABSTRACT

An accurate evaluation of the ultimate bearing capacity of a cylindrical foundation is crucial for predicting pipe–soil interaction behaviors. A general slip-line field solution is derived for the ultimate bearing capacity of a pipeline on the drained soil obeying Mohr–Coulomb criterion. The slip-line field around the pipeline matches well with the corresponding plastic incremental-displacement field simulated by utilizing finite element analysis. Parametric studies indicate that as the internal friction angle of the soil approaches zero, the derived bearing capacity factors for the pipeline on the drained soil limit to those for the pipeline on the undrained soil obeying Tresca criterion. The bearing capacity factors for a fully-smooth pipeline then limit to those for a conventional rectangular strip-footing while the pipeline embedment approaches zero. Moreover, the dimensionless collapse load increases with increasing the pipeline embedment and the pipe–soil interfacial friction.

1. Introduction

The ultimate bearing capacity of a foundation is the pressure causing shear failure of the supporting soil immediately below and adjacent to the foundation (see Knappett and Craig (2012)). Unlike conventional rectangular strip-footings, quite a few subsea structures, e.g., submarine pipelines, risers and mooring lines, hold circular sections. An efficient evaluation of the ultimate bearing capacity of a cylindrical foundation is crucial for predicting subsea structure-soil interaction behaviors, which may significantly affect the on-bottom stability of a submarine pipeline, the configuration of a steel catenary riser (SCR) at its touchdown zone, or the embedment of a circular mooring line into the seabed, etc. In the offshore engineering practices, the possibility for excessive settlement/sinking or floatation of such a subsea foundation should be checked in the design and maintenance stages (Det Norske, 2010).

The ultimate bearing capacity of a conventional shallow foundation with flat bottom on land has been investigated by applying slip-line (stress) field theory and upper-bound theorem of classical plasticity theory, revealing the failure mechanisms (Chen and Liu, 1990; Gourvenec and Randolph, 2003; Li et al., 2015). The bearing capacity of a strip-footing can be treated as a plane-strain problem. For a conventional strip-footing, Prandtl's solution has been widely adopted to predict its bearing capacity. For the submarine pipeline holding a circular cross-section, some researchers, e.g. Small et al. (1971), predict its bearing capacity by the Prandtl's solution with some empirical corrections, i.e. using an equivalent width for certain value of the pipeline embedment. Nevertheless, this simplified treatment obviously could not well consider the effect of the circular section of pipeline foundations. Recently, the slip-line field solution for bearing capacity of a pipeline on clayey soils obeying Tresca failure criterion was derived by Gao et al. (2013). The parametric study indicated that the effect of circular section configuration on the bearing capacity factor gets more obvious with increasing dimensionless pipeline embedment (e_0/r, where e_0 is the pipeline embedment into the seabed, and r is the radius of the pipeline). That is, with the pipeline embedment e_0/r increasing from zero (i.e. the pipeline just touching soil surface) to 1.0 (i.e., the pipeline being half buried), the bearing capacity factor for cohesion (N_c) decreases from the value of "$2+\pi$" to "4.0" accordingly. As such, if the pipeline foundation (with a circular-bottom) is simplified as a conventional strip footing (with a flat-bottom) without any corrections would over-evaluate the bearing capacity.

If undrained bearing capacity is being considered, the soil can be assumed to behave as a Tresca material; if drained bearing capacity is under investigation, the soil can then be regarded as a Mohr–Coulomb material (see Potts and Zdravković (2001)). Previous analytical solutions for the vertical bearing capacity of pipeline foundations are mainly for the purely cohesive soils

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obeying Tresca criterion, e.g., the upper and lower bound solutions by Murff et al. (1989); a slip-line field solution by Gao et al. (2013). Based on the plasticity theory and series of sideswipe tests of a partially embedded pipeline on calcareous sands, several pipe–soil interaction models (or named as force-resultant plasticity models) for the combined vertical and horizontal loading conditions have been successively developed and employed for simulating the pipeline on-bottom responses (e.g., Zhang et al., 2002; Tian and Cassidy, 2008; Hodder and Cassidy, 2010; Tian et al. 2010). As for the aforementioned pipe–soil interaction models, the behaviors of the entire pipe foundation were encapsulated by relating the resultant forces to the corresponding displacements. In the existing sideswipe or penetration tests, the loads on the pipe foundation were normally not beyond its vertical bearing capacity. The vertical bearing capacity is a critical value of the pressure inducing sudden settlement/collapse of the pipe foundation while penetrating into the soil, which is crucial for the vertical on-bottom stability of a submarine pipeline.

In this study, a general slip-line field solution is derived for the ultimate vertical bearing capacity of a pipeline on the drained soil obeying Mohr–Coulomb criterion, taking into account both the cohesion and the internal friction angle of the soil. The slip-line field for a pipeline on Mohr–Coulomb soils is constructed. Parametric studies are further conducted for understanding the failure mechanism of the pipeline foundation on drained soils.

2. A general slip-line solution for the bearing capacity of A pipeline foundation

2.1. Construction of the slip-line field for a pipeline on Mohr–Coulomb soils

For a submarine pipeline laid on a topographically flat seabed, its length is typically much larger than the section dimension, thus the pipeline's bearing capacity is usually treated as a plane-stain problem. As aforementioned, the slip-line field for the cylindrical foundation or pipeline on Treaca soils has been constructed (Gao et al., 2012, 2013). In this section, a general slip-line field for the pipeline foundation on a Mohr–Coulomb soil will be further constructed.

As shown in Fig. 1, the pipeline with radius r is laid on a seabed with an embedment e_0 and uniform surcharge pressure q on the ground surface adjacent to the pipeline. Similar to the previous treatment for the surcharge pressures, in this study, the surcharge pressures (q) are set as follows: (1) for the case of $e_0/r \le 1$, q is set to zero; (2) for the case of $e_0/r > 1$, the pipeline embedment can be treated as $e_0/r = 1$ with an equivalent uniform surcharge pressure $q = \gamma'(e_0 - r)$, where γ' is the submerged (buoyant) unit weight of the soil.

In the analytical study, the seabed is assumed as a homogenous, isotropic and perfectly-plastic material, obeying Mohr–Coulomb

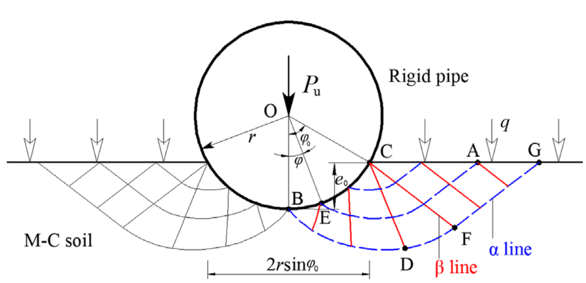

Fig. 1. The slip-line field of the pipeline foundation on a soil obeying Mohr–Coulomb failure criterion: a smooth pipe; internal frictional angle $\phi = 15^\circ$.

failure criterion. As well known, the Tresca criterion/model is for total stress analysis, and the Mohr–Coulomb criterion is for effective stress analyses. If the soil cohesion (c) is replaced with the undrained shear strength (e.g., to simulate the undrained behavior of the saturated clayey soils) and the angle of shearing resistance ϕ is set to zero, the Mohr–Coulomb failure criterion can be degenerated to the Tresca criterion (see Potts and Zdravković (1999)).

The slip-line field is divided into three regions: the uniform region CFG, the extrusion region CBD and the transition region CDF (see Fig. 1). The direction of α and β slip-lines can be uniquely determined once the mean stress σ (Note:$\sigma = (\sigma_1 + \sigma_3)/2$) and the shear stress (or normal stress) on an arbitrary direction are given. Under certain boundary conditions, the slip-line field could be constructed using characteristic functions of slip-line.

With the Riemann condition on CG and CEB, the stresses of the region CFG, CBD and CDF can be calculated successively. The soil within the slip-line field is supposed to be in critical failure condition. The direction of the third principle stress is vertical in the uniform region CFG. Given the value of the uniform load q, the stresses in field CFG and along the boundary CF can be solved. The field CBD is the extrusion region. The direction of the first principle stress at line CEB is radial for a purely smooth pipe. Point E at the boundary CEB are connected with CF by α slip-line and the shear stress can be determined if interfacial friction f is properly defined, the details of which is given in Section 2.2. Therefore, the stresses on the boundary CEB is determined using the characteristic functions. Then the stresses in the field CDB and line CD can be obtained, identically using the characteristic functions. Thus, CDF can be determined according to the stresses at CF and CD. By adopting a finite difference approximation, the whole slip-line field can be constructed (as shown in Fig. 1). Noting that, in the construction of the slip-line field for a Mohr–Coulomb soil, the soil cohesion can be arbitrary, i.e. the magnitude of the slip-line field is not related to the value of soil cohesion. The β and α lines in the uniform region (CFG) are straight lines while the α lines are curved in the extrusion region CBD and the transition region CDF.

2.2. Collapse load: slip-line field solution for Mohr–Coulomb soils

The collapse load (P_u) for the ultimate bearing capacity of a pipeline foundation can be expressed with the integral of the stresses along the pipe–soil interface as follows:

$$P_u = 2\int_0^{\varphi_0} r\sigma_{E,y}\, d\varphi. \tag{1}$$

In which $\sigma_{E,y}$ is the vertical component of the pipe–soil interfacial force; φ_0 is the embedment angle $\angle$BOC(see Fig. 1), i.e. $\varphi_0 = \arccos(1 - e_0/r)$; $\varphi = \angle$BOE is an arbitrary angle for the integration along the pipe–soil contact boundary. Based on the slip-line field theory, the stress equation along the α line for the soil obeying Mohr–Coulomb criterion is derived as (see Appendix A):

$$\sigma_0 e^{-2\omega \tan\phi} = \text{const1} \tag{2}$$

As shown in Fig. 1, the points A and E are along the same α line. Thus the following relationships exist for the stresses at point A and E, i.e.

$$\sigma_{0E} e^{-2\omega_E \tan\phi} = \sigma_{0A} e^{-2\omega_A \tan\phi}, \tag{2'}$$

In which, $\sigma_0 = c\cot\phi + \sigma$, where c and ϕ are the cohesion and internal friction angle of the soil respectively; $\sigma (= (\sigma_1 + \sigma_3)/2)$ is the mean stress in the soil; ω is the angle from the x-axis to the first principal stress plane in the clockwise direction (see Fig. 9).

The point-A is along the soil surface, $\omega_A = \pi/2$ and $\sigma_{0A} = (q + c\cot\phi)/(1 - \sin\phi)$. Fig. 2 shows the Mohr-circle for