Ocean Engineering 104 (2015) 422–436

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

A reduced integration method for the coupled analysis of floating production systems

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ARTICLE INFO

Article history:
Received 29 September 2014
Accepted 22 May 2015
Available online 12 June 2015

Keywords:
Floating production systems
Coupled analysis
Nonlinear dynamics
Time integration methods

ABSTRACT

This work presents a reduced integration method for the nonlinear coupled analysis of floating production systems, combining a "strong coupling" formulation with a variation of the Ritz-Wilson basis reduction technique. This leads to an uncoupled formulation for the reduced equations of motion, which may be solved analytically by the Duhamel integral expressing the convolution of the Green functions. These equations are implemented in a recursive time-domain step-by-step solution procedure where the convolution integrals are calculated numerically, with the range of the integration being simply the time step Δt. The method is tailored for the particular characteristics of the nonlinear dynamic behavior of FPS, including the selection of time steps between basis reevaluations: they are triggered according to the natural periods of the horizontal motions. The results from the case studies indicate that the goal of obtaining an efficient solution method has been reached, allowing an accurate evaluation of the response with lower computational costs.

1. Introduction

Historically, the constant increase on deepwater oil production activities has motivated studies on different types of offshore platforms, including fixed compliant structures and moored floating production systems (FPS). Considering the Brazilian case, the assessment of very large deepwater oil fields in the Campos Basin around the 1980s has led to studies on compliant towers (Benjamin et al., 1988), and on semi-submersible platforms or FPSO units (de Souza et al., 1998; Jacob et al., 1999). It is well known that large displacements and severe nonlinear dynamic effects characterize the behavior of such systems; therefore, research efforts have been directed to the development of nonlinear dynamic analysis tools with improved computational efficiency (see for instance Jacob and Ebecken, 1993, 1994a, 1994b).

More recently, with the discovery of large reserves in the pre-salt layer in the Santos Basin, the focus has settled on FPS. The traditional design practice of FPS employed numerical tools based on uncoupled formulations, which did not take into account the nonlinear dynamic interaction between the floating unit and the mooring lines and risers. Presently, in deep-water scenarios it is widely acknowledged that the moored system and the risers comprise a truly integrated system. That is, one should not disregard the coupling between the hydrodynamic behavior of the hull and the structural-hydrodynamic behavior of the mooring lines and risers represented by Finite Element – FE models (Astrup et al., 2001; Chaudhury, 2001; Correa et al., 2002; Correa, 2003; Finn et al., 2000; Heurtier et al., 2001; Kim et al., 2001, 2005; Ormberg and Larsen, 1998; Senra et al., 2002; Wichers and Devlin, 2001). Therefore special attention has been dedicated to the development and implementation of formulations for the coupling of the equations of motion that represents the hull and the lines (see for instance Garrett et al., 2002a, 2002b; Garrett, 2005; Kim et al., 2005; Rodrigues et al., 2007; Tahar and Kim, 2008; Low and Langley, 2006, 2008; Low, 2008).

However, the use of coupled models may still lead to excessive computational costs, and the extensive use of time-domain fully coupled analyses might not be feasible for the current design practice, which requires, for instance, hundreds or thousands of analyses for the assessment of fatigue behavior of the risers. In this context, Jacob et al. (2012a, 2012b) described alternative formulations for the coupled analysis of FPS, and presented parallel implementations associated to those formulations. The goal was to obtain improved computational efficiency while maintaining higher levels of accuracy (attained by a full time-domain method

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http://dx.doi.org/10.1016/j.oceaneng.2015.05.033
0029-8018/

and a rigorous representation of the lines by Finite Element models).

In this work, we continue the search for still more efficient strategies for the coupled analysis of FPS, by employing *basis reduction* techniques comprising a "reduced integration" method. Basis reduction techniques such as the Mode Superposition Method – **MSM** (Bathe, 1996) and the Ritz-Wilson Methods – **RWM** (Wilson et al., 1982) consists on applying a coordinate transformation to the equations of motion, which are then expressed in another vectorial subspace, resulting in a dramatic reduction on the number of unknowns. Such methods have been successfully employed for the solution of linear structural dynamic problems, where the coordinate transformation matrix can be calculated and applied only once, prior to the time integration process. This way, the overhead involved in the coordinate transformation is largely compensated by the lower costs of integrating a much smaller number of equations, leading to an expressive increase in computational efficiency.

Reduction techniques and other substructuring methods have also been successfully employed for inertial problems with mild nonlinearities or with localized nonlinear effects (Bathe and Gracewski, 1981); in these cases the stiffness matrix is never reevaluated, and the coordinate transformation matrix can still be determined only once. However, the benefits of applying a reduced integration method to more severe nonlinear problems may be smaller or even inexistent, mainly because the stiffness matrix (and therefore the transformation matrix) should be frequently updated along the time integration process. Jacob and Ebecken (1992) presented an adaptation of reduction techniques for nonlinear problems, including fixed offshore systems such as jackets or compliant towers. That work proposed an adaptive strategy to minimize the instances of reevaluation of the stiffness and coordinate transformation matrices, allowing the associated overheads to be compensated by the increased computational efficiency of integrating a reduced system of equations between the basis reevaluation steps.

Now, the strategy proposed in this work is to associate the "strong coupling" **StC** formulation described by Jacob et al. (2012a) with extensions of the **RWM** basis reduction technique, comprising a reduced integration method tailored for the particular characteristics of the nonlinear dynamic behavior of FPS. We consider several aspects in this tailoring, including the selection of time steps between basis reevaluations (triggered according to the natural periods that correspond to the horizontal motions of the FPS). The goal is to obtain an efficient solution method for the nonlinear coupled analysis of FPS, allowing an accurate evaluation of the response with lower computational costs.

The remainder of this paper is organized as follows: initially, Section 2 summarizes the equations of motion for the hull and lines, and the corresponding (unreduced) solution methods associated to the two alternative coupling formulations as presented by Jacob et al. (2012a), referred as *weak coupling* **WkC** and *strong coupling* **StC**. Those formulations are based on the simultaneous time-domain integration of the equations of the hull and lines; it will be seen that they are characterized by how these equations are associated and solved.

Section 3 then presents the fundamentals of reduced integration methods in general, and describes the main characteristics of the **RWM**. The main aspects of the reduced integration method tailored for the coupled analysis of FPS are detailed in Section 4: the algorithm for the generation of the Ritz vectors and its termination criteria; the formulation of the reduced uncoupled equations of motion; the recursive procedure devised for their solution, using the Duhamel integral and Green functions; the basis reevaluation strategy; and the criteria for the selection of time-step values.

Case studies are presented in Sections 5 and 6, comparing motions and tensions provided by the reduced integration method (as well as the computational cost) with the corresponding results provided by a direct implementation of the Newmark time integration method that does not perform basis reduction. Section 5 presents a simple, academic case study, while Section 6 presents results of coupled analyses of a FPS representative of actual applications. Finally, based on the results of these studies, Section 7 presents the final remarks and conclusions.

2. Equations of motion; coupling formulations

2.1. Hull: large amplitude, rigid body equations of motion

The rigid-body motions of the hull may be represented by the following set of 12 differential equations, comprising the exact large amplitude equations of motion (Meirovitch, 1970; Paulling, 1992):

$$\frac{d\mathbf{v}}{dt} = \mathbf{M}_h^{-1}\mathbf{f}, \quad \frac{d\mathbf{x}}{dt} = \mathbf{v} \tag{1a}$$

$$\frac{d\boldsymbol{\omega}}{dt} = \mathbf{I}_h^{-1}[\mathbf{m} - \boldsymbol{\omega}\mathrm{x}(\mathbf{I}_h\boldsymbol{\omega})], \quad \frac{d\boldsymbol{\theta}}{dt} = \mathbf{B}^{-1}\boldsymbol{\omega} \tag{1b}$$

The unknowns are $\mathbf{x}$, $\boldsymbol{\omega}$, $\mathbf{v}$ and $\boldsymbol{\theta}$ (respectively translational and angular components of position and velocities of the body as functions of time). $\mathbf{M}_h$ and $\mathbf{I}_h$ are 3×3 matrices with, respectively, the dry mass of the hull and the moments/products of inertia, and $\mathbf{B}$ is a rotation matrix with sines and cosines of the Euler angles.

Vectors $\mathbf{f}$ and $\mathbf{m}$ contain, respectively, external forces and moments due to the environmental loadings of wind, wave and current. The calculation of these loads is associated to the particular hydrodynamic model considered to represent the hull, as described by Jacob et al. (2012a). For platforms based on ships such as FPSOs, a full radiation/diffraction model based on the Potential Theory is available (Chakrabarti, 1987). For platforms comprised by large-diameter members such as Tension-Leg platforms or semi-submersibles, the hull may be represented by a hybrid Morison/Radiation–Diffraction hydrodynamic model (Hooft, 1982; Paulling, 1992). This model allows the representation of diffraction and radiation effects by combining the following hydrodynamic forces: **(a)** The 1st-order forces from the Morison formula, including drag and added-mass inertia forces; **(b)** The 1st-order Froude–Krylov forces; and **(c)** forces from the Potential Theory, including 2nd-order wave diffraction effects (generating mean and slow drift forces) and also radiation effects (generating frequency-dependent radiation damping), both calculated from coefficients previously determined by a hydrodynamic analysis program such as WAMIT (Lee, 1998). As will be seen later, in the **WkC** coupling scheme the vectors $\mathbf{f}$ and $\mathbf{m}$ also include the resultant forces and moments of the lines at the connections with the hull.

Eq. (1) may be solved in the time domain by the fourth-order Runge–Kutta (R–K) time integration method.

2.2. Lines: semi-discrete equations of motion

The lines are represented by FE models; according to the type of line (mooring, flexible or steel catenary risers, umbilical cables, etc.), either nonlinear truss or frame elements can be employed for the spatial discretization. The following form of the equations of