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Stress field and fatigue strength analysis of 135-degree sharp corners based on notch stress strength theory

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ABSTRACT

Fatigue strength is known to be closely related to the precise geometrical discontinuity of the welded joints. It is necessary to better understand the fatigue behavior of welded joints with consideration of the geometrical factors that produce locally high stresses. Based on notch stress strength theory, this paper proposes a simple method using the singularity strength '*as*' to estimate the stress field distribution at the corner. By comparing the expected results of 135° sheet corner with those obtained by N-SIF formula, the contribution of the singularity strength '*as*' is quantified for different geometries, and summarized into concise expressions. Subsequently, the stress field parameters, namely the notch stress intensity factors (N-SIFs), are combined with '*as*' formula to predict the fatigue strengths of cruciform weldments. Validity of this method is further verified by analyzing the fatigue test results for several cruciform welded joints reported in literatures. The results show that the proposed '*as*' method has the additional advantage in simplified stress field and fatigue strength estimation for welded joints.

1. Introduction

Geometrical discontinuities in mechanical components and structures provoke, as is well known, more or less localized concentration of stress field. In the ship and ocean engineering, the simplest type of plate girder structure consists of plates, stiffeners and brackets. Stress concentrations at these corners cause cracks that initiate and propagate under cyclic loadings. Knowledge of such stress distributions in the vicinity of stress raisers is frequently required for an accurate design of the component (Lazzarin and Tovo, 1996).

Sharp notches stress field in the proximity of crack initiation points is an intriguing problem, particularly when the analysis aim is to establish a direct link between stress level and the fatigue strength of welded components (Atzori et al., 1999). Williams (1952) pointed out that, according to the elasticity theory, the asymptotic stress near a re-entrant corner is singular and the singularity degree just depends on the notch opening angle. Moreover, the SIF is a function of the overall geometry and the far-field stress. Subsequently, a series of researches have been undertaken to determine the stress field present at various notch geometries and at the singularity points where the interface between two free edges. Bogy (1968, 1971) systematically studied the order of stress singularity. The effect of the opening angle between the interface and free edge on the nature of stress singularity was discussed in his investigations. Gross and Mendelson (1972) focused on the definition and evaluation of generalized SIFs. They used SIFs to describe the V-shaped sharp corner stress field and considered the case of angle $\theta = 0$ as the N-SIFs. In order to describe the influence of the stress distributions on fatigue behavior close to weld toes, Lazzarin and Tovo (1998) quantified the contributions of the symmetric and skew-symmetric stress modes. The numerical results were summarized into expressions which also took into account the influence of the main geometrical parameters of the welded joint. However, the quantitative formula is not very straightforward and convenient. Gao et al. (2013) and Xu et al. (2013) presented the simple prediction formula applicable to notch stress intensity factors K_1^N (for opening mode), but neglected the influence of K_2^N (for sliding mode). However, it is not reasonable to neglect the influence of K_2^N (Lazzarin and Tovo, 1998). According to these previous studies, further researches should be implemented to simplify these problems.

Meanwhile, the traditional *S–N* curve approach is based on the design nominal stress without considering the effect of stress discontinuity explicitly. The *S–N* curve approach is also difficult to apply directly when the object detail is complicated and incomparable to any classified joints, or the loadings are complex (Xiao and Yamada, 2004). It has already been proved in literature (Lazzarin and Tovo, 1996; Lazzarin and Livieri, 2001) that N-SIFs can be used to predict the fatigue strength of notch components weakened by V-shaped re-entrant corners, where the singularity in the stress distribution makes any failure criterion based on elastic peak stress no longer applicable.

According to Lazzarin and Tovo's notch stress strength theory, this paper proposed a simple method using the singularity strength '*as*' to estimate the stress field distribution at the corner.

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The results are also used to describe and predict sharp corners fatigue strength, both for tensile specimens and bending specimens.

2. Singular stress fields analysis

2.1. Williams method

Williams (1952) stated that, even in a sharp open notch, the stress field is singular close to the tip and the singularity exponent is related to the notch opening angle. In a polar frame of reference(r,θ), the stress field around the corner is the summation of Mode I (opening type) and Mode II (sliding type) stress.

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}=\lambda_1 r^{\lambda_1-1}a_1\begin{Bmatrix}f_{1,\theta}(\theta)\\ f_{1,r}(\theta)\\ f_{1,r\theta}(\theta)\end{Bmatrix}+\lambda_2 r^{\lambda_2-1}a_2\begin{Bmatrix}f_{2,\theta}(\theta)\\ f_{2,r}(\theta)\\ f_{2,r\theta}(\theta)\end{Bmatrix}\tag{1}$$

where r is the distance to the corner, coefficient$a$ is the complex constant, $f_i(\theta)$ are the stress functions, and λ_i are the eigenvalues defined in Eq. (2). For details of $f_i(\theta)$, please see Eq. (3).

$$\sin(\lambda_i q\pi)+\lambda_i\ \sin(q\pi)=0\tag{2}$$

where 'q' is related to the opening angle 2α by means of the expression $2\alpha=\pi(2-q)$ (see Fig. 1). Accordingly, symbols around the 135° corner can be defined as shown in Fig. 2.

Then, stress components for mode I (tension) are given by Lazzarin and Tovo, (1998) as follows:

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}_{\rho=0}=\lambda_1 r^{\lambda_1-1}a_1\left[\begin{Bmatrix}(1+\lambda_1)\cos(1-\lambda_1)\theta\\ (3-\lambda_1)\cos(1-\lambda_1)\theta\\ (1-\lambda_1)\sin(1-\lambda_1)\theta\end{Bmatrix}+\chi_1(1-\lambda_1)\begin{Bmatrix}\cos(1+\lambda_1)\theta\\ -\ \cos(1+\lambda_1)\theta\\ \sin(1+\lambda_1)\theta\end{Bmatrix}\right]\tag{3}$$

For mode II (shear):

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}_{\rho=0}=\lambda_2 r^{\lambda_2-1}a_2\left[\begin{Bmatrix}(1+\lambda_2)\sin(1-\lambda_2)\theta\\ (3-\lambda_2)\sin(1-\lambda_2)\theta\\ (1-\lambda_2)\cos(1-\lambda_2)\theta\end{Bmatrix}+\chi_2(1+\lambda_2)\begin{Bmatrix}\sin(1+\lambda_2)\theta\\ -\ \sin(1+\lambda_2)\theta\\ \cos(1+\lambda_2)\theta\end{Bmatrix}\right]\tag{4}$$

In order to give a physical meaning to the constant values present in Williams' formula, Gross and Mendelson (1972) proposed to extend the definition of the SIF, commonly used to describe crack stress fields, to open notches. Based on the stress field components, the definitions for N-SIFs are as follows:

$$K_1=\sqrt{2\pi}\lim_{r\to0}(\sigma_\theta)_{\theta=0}r^{1-\lambda_1};\quad K_2=\sqrt{2\pi}\lim_{r\to0}(\tau_{r\theta})_{\theta=0}r^{1-\lambda_2}\tag{5}$$

When the opening angle$2\alpha=0$,K_1 equals to the stress intensity factor K_I of the linear elastic fracture mechanics. Applying definitions (5) to (3) and (4), the real and imaginary parts of the complex constant a are

$$a_1=\frac{K_1}{\lambda_1\sqrt{2\pi}[(1+\lambda_1)+\chi_1(1-\lambda_1)]};\quad a_2=\frac{K_2}{\lambda_2\sqrt{2\pi}[(1-\lambda_2)+\chi_2(1+\lambda_2)]}\tag{6}$$

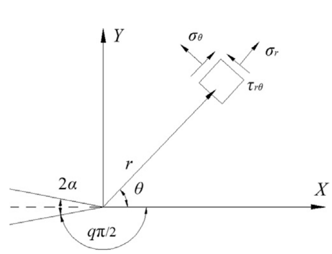

Fig. 1. Symbols of sharp V-shaped notch stress field.

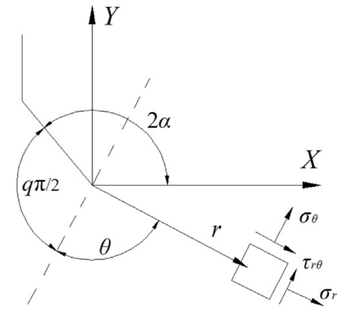

Fig. 2. Symbols around the 135° corner.

It is therefore possible to present Williams' formula for stress components as a function of the N-SIFs for Mode I, tension as follows:

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}_{\rho=0}=\frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_1-1}K_1}{[(1+\lambda_1)+\chi_1(1-\lambda_1)]}\left[\begin{Bmatrix}(1+\lambda_1)\cos(1-\lambda_1)\theta\\ (3-\lambda_1)\cos(1-\lambda_1)\theta\\ (1-\lambda_1)\sin(1-\lambda_1)\theta\end{Bmatrix}+\chi_1(1-\lambda_1)\begin{Bmatrix}\cos(1+\lambda_1)\theta\\ -\ \cos(1+\lambda_1)\theta\\ \sin(1+\lambda_1)\theta\end{Bmatrix}\right]\tag{7}$$

For Mode II, shear:

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}_{\rho=0}=\frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_2-1}K_2}{[(1-\lambda_2)+\chi_2(1+\lambda_2)]}\left[\begin{Bmatrix}(1+\lambda_2)\sin(1-\lambda_2)\theta\\ (3-\lambda_2)\sin(1-\lambda_2)\theta\\ (1-\lambda_2)\cos(1-\lambda_2)\theta\end{Bmatrix}+\chi_2(1+\lambda_2)\begin{Bmatrix}\sin(1+\lambda_2)\theta\\ -\ \sin(1+\lambda_2)\theta\\ \cos(1+\lambda_2)\theta\end{Bmatrix}\right]\tag{8}$$

where $K_i=\sigma_0\cdot k_i\cdot t^{1-\lambda_i}$ are N-SIF values; $k_i=f(h,t,L)$is a geometric parameter; and σ_0 is the nominal tensile or bending stress.

2.2. Modified singularity strength method

According to the above equations, the axial stress distribution of Mode I at a corner can be described as

$$\sigma_{\theta1}=\frac{\sigma_0}{\sqrt{2\pi}}\cdot\frac{1}{x^{p_1}}\cdot[C_1(\alpha,\theta)\cdot f_1(h,t,L)^{1/p_1}\cdot t]^{p_1}\tag{9a}$$

And for Mode II

$$\sigma_{\theta2}=\frac{\sigma_0}{\sqrt{2\pi}}\cdot\frac{1}{x^{p_2}}\cdot[C_2(\alpha,\theta)\cdot f_2(h,t,L)^{1/p_2}\cdot t]^{p_2}\tag{9b}$$

where x is the distance to the corner;

$$C_1(\alpha,\theta)=\left[\frac{(1+\lambda_1)\cos(1-\lambda_1)\theta+\chi_1(1-\lambda_1)\cos(1+\lambda_1)\theta}{(1+\lambda_1)+\chi_1(1-\lambda_1)}\right]^{1/p_1};$$

$$C_2(\alpha,\theta)=\left[\frac{(1+\lambda_2)\sin(1-\lambda_2)\theta+\chi_2(1+\lambda_2)\sin(1+\lambda_2)\theta}{(1-\lambda_2)+\chi_2(1+\lambda_2)}\right]^{1/p_2};$$

$f_i(h,t,L)=k_i$, k_i is a function of h,t,L;

$p_i=1-\lambda_i$, for different corner angles, from Williams (1952), are shown in Table 1. Parameters are defined in Fig. 2.

For a given corner (2α) at a given direction (θ), as shown in Table 1, $p_i$and $C_i(\alpha,\theta)$ are constants; k_i is a function of h,t,L. To simplify the calculation of $C_i(\alpha,\theta)\cdot f_i(h,t,L)^{1/p_i}\cdot t\cdot\pi^{-(1/2p_i)}$, a new parameter '$as$' is introduced and named as singularity strength to describe the stress field at the corner, for Mode I as follows:

$$\sigma_{\theta1}=\frac{\sigma_0}{\sqrt{2}}\left(\frac{as_1}{x}\right)^{p_1}\tag{10a}$$