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Short Communication

# Free vibration analysis of a delaminated beam–fluid interaction system

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A delaminated offshore structure having the form of a laminated composite beam partially contacting with a fluid is considered and its free vibration characteristics are presented in this paper. The delaminated beam is modeled as a uniform Bernoulli–Euler cantilever beam fixed at the bottom. The beam is analyzed as five interconnected sub-beams in which the continuity and compatibility conditions are satisfied between adjoining beams. Transverse displacements of the beam are approximated by a set of admissible trial functions which are required to satisfy the boundary, continuity and compatibility conditions. Based on the eigen-value technique, the solution for the natural frequencies and the corresponding mode shapes are obtained. The numerical results have been compared with the existing results and a close agreement is achieved. Furthermore, the effect of different parameters including free surface wave, the fluid density and material anisotropy on the natural frequencies of the delaminated beam based on the free and constrained modes are examined and discussed in detail.



## 1. Introduction

Laminated composites are widely used in many offshore structures such as a dam or a tower surrounded by a fluid. This is due to their superior characters such as high strength–stiffness, light weight, fatigue resistance etc. Contrary to these mechanical merits, they are subjected to a wide range of damages induced during their fabrication or service life which may significantly reduce their structural performance. One of the commonly encountered types of defects or damages in the multi-layered composite structures is fluid boundaries.

Vibration response of a dry delaminated beam has received a good amount of attention in the literature (Wang et al., 1982; Mujumdar and Suryanarayan, 1988; Tracy and Pardoen, 1989; Hu and Hwu, 1995; Shu and Fan, 1996; Kargarnovin et al., 2013a, 2013b), whereas vibration response of a wet delaminated beam has not been investigated yet. For convenience, the beam in contact with fluid is called the *wet beam*, and the dry beam indicates the beam in contact with air. It is of practical value for structural engineers involved in the dynamic analysis and design of beams surrounded by a fluid. Short fluid boundariess which do not significantly degrade the overall stiffness of the beam have little effect on the dynamic response of the beam, while the presence of a long fluid boundariess significantly changes the response of the beam. This response depends sensitively on the fluid boundaries parameters such as length, spanwise location and depth. Hence the beam response may indicate the presence and the nature of internal fluid boundaries damage.

Wang et al. (1982) examined the free vibrations of an isotropic beam with a through-width fluid boundaries by using four Bernoulli–Euler beams that are connected together. By applying appropriate boundary and continuity conditions, the response of the beam was obtained as a whole. However, the vibration modes are physically inadmissible for off-mid plane fluid boundariess because the delaminated layers were assumed to deform freely without touching each other and thus have different transverse deformations ("*free mode*"). Mujumdar and Suryanarayan (1988) then proposed a model based on the assumption that the delaminated layers are constrained to have identical transverse deformations ("*constrained mode*"). Results from an extensive experimental investigation were also presented which show better agreement with the analytical results obtained by using the constrained mode. The constrained mode model was also used by other researchers such as; Tracy and Pardoen (1989) on a simply supported composite beam, Hu and Hwu (1995) on a sandwich beam, Shu and Fan (1996) on a bimaterial beam, Brandinelli and Massabo (2003)and Kargarnovin et al. (2013a) for dynamic response of a Timoshenko beam under the action of moving force. It is worth to mention that there are many studies in which both the 'free mode' and 'constrained mode' models are used to study the vibration of delaminated beams (Shu and Della, 2004; Della and Shu, 2005; Kargarnovin et al., 2013b).

Westergaard (1933) investigated the hydrodynamic pressure on a beam during earthquakes, although the effect of surface waves was ignored. Chopra (1967) derived an analytical solution of the hydrodynamic pressure on a vertical rigid dam. He developed the theory to

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investigate the hydrodynamic pressure resulting from horizontal and vertical ground motions including the influence of free surface waves. Chopra concluded that the associated contribution of free surface wave disturbance is small. The dynamical behavior of a flexible beam–water interaction system was examined by Xing et al. (1997). The attention was focused on the exact natural frequencies for a unified coupled beam–water dynamic system subjected to a variety of assumptions and boundary conditions in order to assess the influence of such effects on the response. Pani and Bhattacharyya (2009) used a similar approach to develop the dynamic pressure on a plate with the simply supported conditions on all edges and attached to a rigid wall.

Chen and Yang (2006) have investigated the natural frequency and stability of an axially moving viscoelastic beam with hybrid supports, i.e., the moving beam is constrained by simple supports with torsion springs at both ends. Later on, Lin and Qiao (2008) presented the vibration and stability analysis for an axially moving beam in fluid and constrained by simple supports with torsion springs. The equations of motion of the beam with uniform circular cross-section, moving axially in a horizontal plane at a known rate while immersed in an incompressible fluid were derived. An axial added mass coefficient and an initial tension were implemented in these equations. Wu and Hsu (2006) have been presented a unified approach to determine the exact lowest several natural frequencies and the associated mode shapes of the partially or fully immersed beam in both the elastic- and fixed-support conditions. Furthermore, by modeling the distributed added mass along the immersed part of the beam with a number of concentrated added masses, a point added mass method incorporated with the mode-superposition approach was also proposed to determine the approximate lowest several natural frequencies and the associated mode shapes of the last two types of immersed beam. A simple procedure based on an empirical added mass formulation and Rayleigh–Ritz method was introduced by Liang et al. (2001) to determine the vibration frequencies and mode shapes of a submerged cantilever thin plate. Wu and Chen (2003) studied the dynamic analysis of the immersed beams with non-uniform cross-sections carrying multiple concentrated attachments using the point added mass method. Zhao et al. (2002) and Xing (2007) developed analytical formulations to examine the dynamic response of a cantilever flexible beam interacting with a 2D semi-infinite water domain, and discussed the effects of various boundary conditions of the fluid domain. Most recently, Bouaanani and Benjamin (2015) proposed an efficient simplified method to determine the modal dynamic and earthquake response of coupled flexible beam–fluid systems and to evaluate their natural vibration frequencies.

The review of the published papers clearly indicates that there are no investigations in the literature on the dynamic response of the wet delaminated beam. Thus, the main contribution of the present work is on the dynamic analysis of the delaminated beam surrounded by a fluid. To do this, first the vibration of a laminated composite beam (LCB) with single fluid boundaries partially contacting with a fluid is formulated. The formulations are derived based on the free and constrained mode models. The developed methodology extends available analytical solutions for the natural vibration frequencies and corresponding mode shapes of the slender delaminated composite beam to include the effects of fluid–structure interaction. Secondly, the natural frequencies based on the present model are verified with available results. Finally, the vibration characteristics of the delaminated LCB in contact with a fluid are presented.

## 2. Problem formulation

Consider the coupled delaminated LCB-fluid interaction system as shown in Fig. 1(a). The beam has the length of $L$, rectangular cross-section of $b\times h$ and wet height $H$. There is an embedded fluid boundaries at the depth of $h_2$ from the free surface with length of $L_2$ located at $L_1$ with respect to the lower support. To study the dynamics of such beam, classical beam theory is employed.

As it can be seen from Fig. 1(b), the beam can be modeled as a combination of five sub-beams connected at the fluid boundaries/fluid boundaries $x=L_1$, $x=L_1+L_2$ and $x=L_1+L_2+L_4$. In this way, there will be five sub-beams of one to five with lengths and thicknesses of $L_i\times h_i$ $(i=1$ to $5)$ where $L_2=L_3$, $L_4=H-L_1-L_2$, $L_5=L-H$, $h_1=h_4=h_5=h$ and $h_3=h-h_2$, respectively.

### 2.1. Assumption and equations of motion

The governing equations of motion for the delaminated LCB contacting with a fluid are obtained based on the 'free' and 'constrained' modes. The fluid pressure satisfying the fluid boundary conditions is used in the above mentioned equations. By employing the eigenvalue technique, the natural frequencies and corresponding mode shapes are derived.

#### 2.1.1. Free mode

The equations of motion for each of the sub-beams are as follows:

$$D_i w_{i,xxxx} + m_i w_{i,tt} = -p(x,z,t)\big|_{z=0}, \quad (i=1,3,4) \tag{1-a}$$

$$D_i w_{i,xxxx} + m_i w_{i,tt} = 0, \quad (i=2,5) \tag{1-b}$$

in which $w$ is the beam's deflection, $m_i$ and $D_i$ $(i=1\text{–}4)$ are denoted as the mass density per unit length and reduced bending stiffness, respectively. Moreover, $p(x,z,t)$ indicates the fluid pressure distribution function. On the interface between the wet beam and the

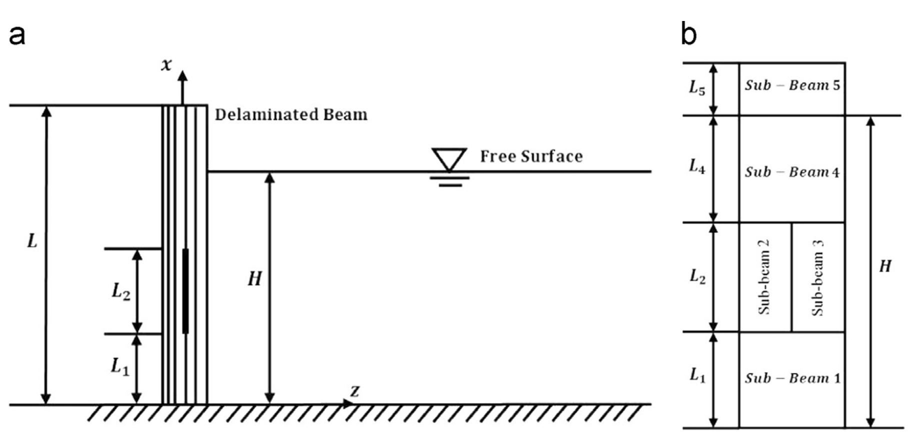


**Fig. 1.** (a) The coupled delaminated LCB-fluid interaction system; (b) the delaminated LCB is modeled by five interconnected sub-beams.