Ocean Engineering 101 (2015) 67–77

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Tsunami propagation over varying water depths

Taemin Ha [a,1], Yong-Sik Cho [b,*]

[a] Department of Civil Engineering, Kangwon National University, 346 Joongang-ro, Samcheok 245-711, Republic of Korea
[b] Department of Civil and Environmental Engineering, Hanyang University, 222 Wangsimni-ro, Seongdong-gu, Seoul 133-791, Republic of Korea

ARTICLE INFO

Article history:
Received 1 September 2014
Accepted 5 April 2015
Available online 22 April 2015

Keywords:
Tsunami
Boussinesq equations
Numerical simulation
Numerical dispersion

ABSTRACT

The linear Boussinesq equations are an ideal model for transoceanic propagation of tsunamis. However, they are impractical for real-time application because Boussinesq-type equation models rely on a fine grid system and therefore require a huge computational domain. Thus, shallow-water equations models are the preferred method of predicting propagation and run-up of near- and far-field tsunamis since they produce fairly accurate results with a much smaller computational requirement. There may be an additional benefit in including physical dispersion effects in numerical models since shallow-water equations theoretically neglect the effect of dispersion on the transoceanic propagation of tsunamis. In this study, a modified finite difference scheme was proposed that adds terms to the linear shallow-water equations in order to account for varying water depths. The proposed model was verified by applying it to tsunami propagation over a submerged shoal and the results were compared with those of the well-known Boussinesq equations model, FUNWAVE. The proposed model was further tested by simulating transoceanic tsunami propagation on real topographies and comparing the numerical results with available observed data.

1. Introduction

Tsunamis can be triggered by impulsive ground motions such as undersea earthquakes, landslides, or meteorite impacts. A tsunami generated by an undersea earthquake will travel in all directions from its point of origin and, upon approaching a coastal area, the shoaling effect will cause it to increase in amplitude. This phenomenon causes severe damage to seaside towns and coastal zone facilities.

The occurrence frequency of devastating tsunamis has markedly increased over the last several decades. On March 11, 2011, the East Japan Earthquake generated a tsunami that hit the Pacific coast of Japan along the Japan Trench zone. The tsunami has been regarded as the largest in Japan's recorded history (Hayashi et al., 2011), and it was particularly devastating because it originated from a location very near coastal communities and made landfall before area residents had time to evacuate. The number of deaths and missing persons reached almost 20,000, and the property damage totaled more than 300 billion USD including Japan, Chile, Hawaii and California coast in USA according to the NOAA website (http://www.ngdc.noaa.gov/hazard/honshu_11mar2011.shtml). Post-tsunami surveys of run-up and inundation reported that the maximum run-up reached approximately 40 m in the Tohoku region (Mori et al., 2011).

It is essentially impossible to forecast an undersea earthquake that may trigger a tsunami. However, tsunami damage may be mitigated through alternative means, such as a tsunami warning system. A properly established warning system must be based on numerical modeling that accounts for multiple compounding variables, such as tsunami generation, propagation, and run-up processes, and that predicts important information including arrival time and run-up heights. Thus, the development of an accurate and efficient numerical model for computing tsunami propagation is paramount in tsunami preparation and damage mitigation.

Over the past several decades, multiple numerical models for simulating transoceanic tsunami propagation have been developed. According to Kajiura and Shuto (1990), frequency dispersion plays an important role in tsunami propagation over long distances. On the other hand, the small wave slope of a typical tsunami results in an insignificant nonlinear advective inertia force that can be disregarded (Imamura et al., 1988). Therefore, the linear Boussinesq equations, which include weak dispersion effects, should accurately simulate the transoceanic propagation of tsunamis (Imamura et al., 1988). However, due to higher-order dispersion terms that arise during the modeling of long-distance propagation, the Boussinesq equation models must be processed with finer spatial and temporal resolutions and higher-order numerical algorithms to avoid numerical dispersion and truncation errors that can affect the accuracy of model results. These finely resolved grids increase the computational expense of the

* Corresponding author. Tel.: +82 2 22200393; fax: +82 2 22201945.
E-mail addresses: tmha@kangwon.ac.kr (T. Ha), ysc59@hanyang.ac.kr (Y.-S. Cho).
[1] Tel.: +82 33 5706514; fax: +82 33 5706517.

http://dx.doi.org/10.1016/j.oceaneng.2015.04.006
0029-8018/

Boussinesq equation models, and are generally overcome by using implicit schemes that enhance computational stability.

Recent advancements in computer technology have allowed for the direct application of the Boussinesq-type equation for tsunami simulations and case studies (Grilli et al., 2007, 2010; Ioualalen et al., 2007; Park et al., 2013; Lynett et al., 2012a, 2012b). Although running such models over large, ocean basin-scale grids with sufficiently fine resolution is no longer unattainable, it still requires very high computational costs such as a parallel computing system (Grilli et al., 2013; Ha et al., 2014). The computation time required for modeling with the Boussinesq-type equations is a severe limitation in their use in a time-sensitive tsunami warning system. Since tsunamis travel quickly and can cause extensive damage to coastal regions in a very short time, brief but accurate computation is necessary for an early tsunami warning system. Furthermore, a tsunami hazard map, drawn using repeated virtual tsunami simulations, could be useful in saving lives, but would require a large time investment to analyze enough scenarios with the Boussinesq-type equations. Therefore, the shallow-water equation models are generally preferred by scientists and engineers to simulate tsunami propagation and run-up because they offer efficient computation capability without sacrificing too much accuracy. These shallow-water equations models have been used in many successful tsunami case studies (Cheung et al., 2011; Apotsos et al., 2011; Kilinc et al., 2009; Nandasena et al., 2012). Among others, the new version of COMCOT (COrnell Multigrid COupled Tsunami Model), a well-known tsunami model based on the shallow-water theory with improved dispersion effects (Wang and Liu, 2011), has been released recently and applied to various tsunami cases (Chai et al., 2014; Lynett et al., 2012a,b; Son et al., 2011; Wijetunge, 2012).

Imamura et al. (1988) presented a finite difference model for the simulation of transoceanic tsunami propagation using the shallow-water equations models. Their model solves the linear shallow-water equations using the explicit leap-frog scheme on a staggered-grid system. The linear shallow-water equations do not address frequency dispersion, but related terms are taken into account by adjusting the numerical dispersion. However, the frequency dispersion effects that are diagonal to the principle axes of the computational domain are not properly represented in the algorithm. Cho (1995) improved the numerical algorithm to properly include frequency dispersion effects in all directions of tsunami propagation. However, the grid size needs to be locally adjusted according to the time step and the local water depth, and therefore this approach is difficult to implement. Cho et al. (2007) proposed a modified scheme that achieves improved accuracy by addressing the spatial-grid and temporal-step size limitations using dispersion-correction terms. However, these improved numerical models were developed using governing equations derived by assuming a constant water depth, and therefore they produced numerical errors when they were used to simulate transoceanic tsunami propagation over the real-sea bathymetry, where water depth varies continuously.

In this paper, a novel, modified dispersion-correction scheme was presented that introduced additional terms to the scheme proposed by Cho et al. (2007). The governing equations were derived based on an assumption of varying water depth, and a numerical scheme describing distant propagation of tsunamis was proposed. The proposed numerical scheme corrects for the Gaussian shape of the free surface, and wave propagation over a submerged shoal is considered. The predicted results were compared with those obtained using FUNWAVE (Wei et al., 1995) to verify their accuracy, and they were then applied to case studies in real topographies: the Central East Sea Tsunami on May 26, 1983 and the Tohoku Tsunami on March 11, 2011.

2. Governing equations

2.1. Linear Boussinesq equations

The propagation of tsunamis over long distances is significantly influenced by frequency dispersion, which must be accounted for to achieve accurate simulation of tsunami propagation. Nonlinear convective inertia force, however, is not a significant variable and can be excluded in propagation modeling. Thus, the Boussinesq equations, which include dispersion terms, are more accurate governing equations for simulations of transoceanic tsunami propagation than conventional shallow-water equations that do not contain dispersion terms (Imamura et al., 1988; Liu et al., 1994). The linear Boussinesq equations are:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}=0 \tag{1}$$

$$\frac{\partial P}{\partial t}+gh\frac{\partial \zeta}{\partial x}=\frac{1}{2}\frac{\partial}{\partial t}\left[h^2\left(\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 Q}{\partial x \partial y}\right)-\frac{h^3}{3}\left\{\frac{\partial^2}{\partial x^2}\left(\frac{P}{h}\right)+\frac{\partial^2}{\partial x \partial y}\left(\frac{Q}{h}\right)\right\}\right] \tag{2}$$

$$\frac{\partial Q}{\partial t}+gh\frac{\partial \zeta}{\partial y}=\frac{1}{2}\frac{\partial}{\partial t}\left[h^2\left(\frac{\partial^2 P}{\partial x \partial y}+\frac{\partial^2 Q}{\partial y^2}\right)-\frac{h^3}{3}\left\{\frac{\partial^2}{\partial x \partial y}\left(\frac{P}{h}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{Q}{h}\right)\right\}\right] \tag{3}$$

where ζ is the free-surface displacement; h is the still-water depth; P and Q are the depth-integrated volume fluxes in the x- and y-axis directions, respectively; and g denotes the acceleration of gravity. Eqs. (2) and (3) include the dispersion terms on the right-hand side. Since dispersion terms can contain both high-order derivatives and mixed space–time derivatives, equations containing these terms are very difficult to use for tsunami propagation simulations.

Assuming water depth is constant, the linear Boussinesq equations can be simplified by employing the long wave approximation (Liu et al., 1994):

$$\frac{\partial^2 \zeta}{\partial t^2}-gh\left(\frac{\partial^2 \zeta}{\partial x^2}+\frac{\partial^2 \zeta}{\partial y^2}\right)=\frac{gh^3}{3}\left(\frac{\partial^4 \zeta}{\partial x^4}+2\frac{\partial^4 \zeta}{\partial x^2 \partial y^2}+\frac{\partial^4 \zeta}{\partial y^4}\right) \tag{4}$$

Cho et al. (2007) developed a numerical model for simulating tsunami propagation using the above equation. They replaced the physical dispersion of the linear Boussinesq equations with modified numerical dispersion, which was generated by discretization of the shallow-water equations using the leap-frog scheme. However, their governing equation was derived using a constant water depth, and therefore, their model could not be applied to rapidly varying topographies.

In the present study, the governing equation was derived using a first-order equation in horizontal coordinates to represent water depth, resulting in a bottom slope with the following coefficients:

$$h=ax+by+c \tag{5}$$

$$\frac{\partial h}{\partial x}=a,\ \frac{\partial h}{\partial y}=b \tag{6}$$

where a and b are calculated under local approximation and treated as constants in deriving Eq. (7). The present model used an explicit method and numerical algorithm including a bottom slope was calculated using locally 5 adjoined grid points [using (i,j), $(i+1,j)$, $(i-1,j)$, $(i,j+1)$, $(i,j-1)$ points for a given (i,j) point] at each time step. In real topography, bottom slopes were not drastically changed in locally 5 grid points, and thus those could be treated as locally constant in the governing equations. Then, Eqs. (5) and (6) allow the linear Boussinesq equation to be reduced to:

$$\frac{\partial^2 \zeta}{\partial t^2}-g\left(a\frac{\partial \zeta}{\partial x}+b\frac{\partial \zeta}{\partial y}\right)-gh\left[\left(2a^2+b^2+1\right)\frac{\partial^2 \zeta}{\partial x^2}+2ab\frac{\partial^2 \zeta}{\partial x \partial y}+\left(2a^2+b^2+1\right)\frac{\partial^2 \zeta}{\partial y^2}\right]$$