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A desingularized Rankine source method for nonlinear wave–body interaction problems

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ABSTRACT

A two-dimensional nonlinear wave–body interaction problem is solved by a desingularized integral method in combination with a mixed Euler–Lagrange method. A special treatment of the intersection point singularity is introduced by employing an optimal technique to smooth the wave elevation around the intersection point and the utilization of a free surface control point distribution. By this means the nonlinear boundary effects arising from body surface and free surface are taken into consideration in addition to the development of a free surface Rankine source distribution method. A Lagrangian formulation is applied to capture the time-dependent motion of the control points and source points to describe the body and free surface nonlinear boundary conditions. The validation of the proposed method is demonstrated by comparing findings with a selection of existing numerical and experimental data.

1. Introduction

The Rankine source method in association with a desingularized method and a mixed Euler–Lagrange method formulates an approach to solve wave–body interaction problems avoiding free surface singularities and permits description of a range of nonlinear wave motion problems. The Euler–Lagrange method was introduced by Longuet-Higgins and Cokelet (1976, 1978) as a time stepping scheme to solve surface wave problems. That is, the field equation is solved through an Euler specification so that the velocity potential is expressed in a boundary integral form and the fluid boundary control points vary with time in the nonlinear boundary problem. This allows description of velocity along the control points in a Lagrange specification.

The far field radiation condition requires careful treatment in the Rankine source method. The infinite free surface integral domain in this method is truncated to a bounded domain but the truncated boundary may cause a reflective wave propagation disturbance. The accuracy and efficiency of numerical results lie in the distributed density of source points and the truncation of the free surface. Several numerical techniques are available to avoid wave reflection, amongst which a numerical beach is widely employed. This method was originated by Israeli and Orszag (1981) who added a Newtonian cooling term and a Reynolds viscosity damping term to the free surface equation. The former acts as a damper absorbing free surface disturbances where the latter modifies the wave dispersion caused by this artificial damping. This method has been further extended to various linear and nonlinear wave–body radiation problems and it is applicable to a large range oscillatory wave frequencies (Sclavounos and Nakos, 1988; Cointe, 1989; Kring, 1994; Nakos et al., 1994; Kring and Sclavounos, 1995; Kim et al., 1997; Huang, 1997). For example, to avoid wave reflection, Kim et al. (1997) employed a kinematic free surface condition equipped with Newtonian cooling and Rayleigh viscosity damping terms, whereas Tanizawa (1996) and Koo and Kim (2004, 2007) added Newtonian cooling to the kinematic free surface boundary condition and introduced damping terms to the dynamic free surface boundary condition. However, the terms added to the free surface boundary conditions give rise to artificially induced numerical errors. In the desingularized method, the free surface domain is divided into inner and outer domains. A fixed number of control points is evenly distributed in the inner domain, whereas in the outer domain neighboring control point distances increase exponentially (Lee, 1992). Due to the periodic characteristic of a wave, the numerical computation can be completed before the surface wave reaches the truncated boundary. This produces a postponement of the wave refection from the truncation boundary and satisfactory results can be obtained (Zhang, 2007).

Previous research adopting this desingularized method either takes account of the nonlinear free surface condition but ignores the nonlinear body surface condition (Cao et al., 1991, 1992, 1993, 1994; Lee, 2003; Beck et al., 1994; Beck and Scorpio, 1995; Scorpio,

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1997; Subramani et al., 1999; Finn, 2003) or otherwise applies the nonlinear body surface condition whilst keeping the linear free surface condition (Zhang and Beck, 2006, 2007, 2008; Bandyk, 2009; Zhang et al., 2010a,b; Bandyk and Beck, 2011). For very large body motions, each of these methods neglect components of the nonlinearity existing in the interaction.

For the nonlinear problem taking account of both nonlinear effects from body surface and free surface, the previously referenced source distribution methods are no longer adequate. In this paper to overcome these limitations, a new frequency dependent control point distribution scheme is developed. Similar to the traditional desingularized method, inner and outer free surface domains are kept. However the control points are no longer distributed evenly in the inner domain. The parameter to determine the exponent in the outer domain is also changed to allow for a larger free surface to be covered. The neighboring control point distances are assessed through a proportionally scheme relating to the top panel size on the body to improve numerical stability and accuracy. Moreover, in the traditional desingularized methods the control points are distributed in the calm water surface, whereas in the present nonlinear desingularized method they are distributed on the nonlinear free surface.

The intersection points of the body and free water surface move in both horizontal and vertical directions as the water surface moves up and down. The singularity at the intersection points, the confluence of body surface and water surface boundaries, generates a surface wave. This singularity can lead to numerical difficulties or even divergence if the intersection points are not properly treated (Kang, 1988; Tsai and Yue, 1996). Vinje and Brevig (1981) applied a body boundary rather than a free surface boundary condition at the intersection points and their positions together with the velocity potential at the intersection points are obtained by extrapolation. The results produced from this approach were not satisfactory. To derive acceptable results from the method, Greenhow et al. (1982) found that it was necessary to use experimental measurements to allocate the intersection points in the process of their computation. Lee (1992) assumed that both body and free surface boundary conditions are satisfied at the intersection points. Similarly, Beck (1994) considered the intersection points as free surface control points and thus free surface boundary conditions are satisfied at the intersection points whereas the body boundary condition is satisfied on the top panels covering the intersection points. A desingularized source is then placed vertically above each of the intersection points. Beck (1994) also discussed the placing of a desingularized control point inside the body, regarding the intersection points as free surface control points. In the present paper, in order to match the new control points distribution, the intersection points are only treated as body panel grid points rather than free surface control points.

In a Lagrange specification, fluid boundary control points are treated as fluid particles. Therefore discrepancies arising from the singularity associated with intersection points rapidly increase and lead to divergent solutions in the numerical scheme. This problem was encountered when a mixed Euler–Lagrange method was first applied and referred to as a sawtooth instability (Longuet-Higgins and Cokelet, 1976, 1978). It was observed that the rate of growth of the instability is independent of the number of time iteration steps, is not influenced by rounding errors and relates to the adopted numerical method and the physical nature of the wave problem. Hence it is necessary to use a smoothing technique (Ferrant, 1997) to the surface wave around the intersection points in the time iteration process.

Longuet-Higgins and Cokelet (1978) introduced a five-point Chebyshev smoothing (filtering) formula to remove the instability for two-dimensional problems and demonstrated the superiority of this five-point approach. Five-point filtering techniques are widely adopted and modified for three-dimensional problems (Xu, 1992; Kring, 1994; Nakos et al., 1994; Kring and Sclavounos, 1995; Kring et al., 1996; Kim et al., 1997; Yan, 2010). Improved results are obtained if higher order polynomial smoothing techniques are employed (Dold, 1992). Koo and Kim (2004) and Zhang et al. (2006) developed a fourth order Chebyshev polynomial method to improve numerical accuracy. The disadvantage of this filtering technique is the occurrence of error spikes although an error spike can be minimized applying the filtering technique, there is a minimum period of application below which the error spike becomes significant and there is a maximum period of application above which the sawtooth instability occurs. Baker et al. (1982) and Lin et al. (1984) utilized a relatively economic smoothing operator technique to identify the iteration divergence. In the present numerical simulation process, we adopt a new smoothing method using the least square principal, which involves only three points. The method is demonstrated to be very efficient in removing the sawtooth instability and error spike.

Due to the application of nonlinear body and free surface boundary conditions, it is necessary to regrid the body surface and free surface control points in both horizontal and vertical directions. In the time iteration process, the number of wetted body surface control points and the number of free surface control points remain constant. Therefore, the size of the influence coefficients matrix is independent of time. As a result of body oscillation, the intersection points are restricted to move in the calm water surface due to the absence of shear force in a potential flow theory. However, the positions of free surface control points and desingularized free surface source points vary vertically and horizontally due to the nonlinear free surface assumption. The force exerted on the wetted body surface depends on the velocity, $\nabla\phi$, and the acceleration potential $\frac{\partial\phi}{\partial t}$. The estimation of $\frac{\partial\phi}{\partial t}$ was discussed by Tanizawa (1995), Bandyk and Beck (2011) and Koo and Kim (2004). In the present paper, the velocity, $\nabla\phi$, on a body surface control point is the time derivative of that control point in the Lagrange specification whereas the acceleration potential $\frac{\partial\phi}{\partial t}$ is calculated by using the total derivative $\frac{d\phi}{dt}$ at a body surface control point in the Lagrange specification and hence $\frac{d\phi}{dt}$ is approximated as a forward difference of the velocity potential with respect to the time variable.

In a three-dimensional high-order Rankine source method using continuous free surface panels, the free surface is discretized as grids of quadrilateral facets and the unknown quantities are represented in the form of biquadratic spline sheets on these facets. In order to comply with this high order spatial discretization, an Emplicit Euler scheme is applied to the temporal discretization process to achieve neutral numerical stability. The free surface kinematic boundary condition is presented in an explicit Euler discretization in time which uses the past time step solutions of fluid velocity to update the velocity. The dynamic boundary condition is satisfied through an implicit Euler discretization which employs the current solution of wave elevation to update the present velocity potential (Sclavounos and Nakos, 1988; Nakos, 1990; Nakos and Sclavounos, 1990; Kring and Sclavounos, 1995; Kim et al., 1997; Huang, 1997). The Rankine source method using the desingularized approach on the free surface simplifies the numerical discretization.

2. Mathematical equations

2.1. Euler formulation

Fig. 1 illustrates a floating symmetric body undergoing forced heave motion in a two-dimensional fluid of infinite depth. The vertical body motion is described by the time-dependent function

$$\xi = a \sin \omega t \tag{1}$$

for an oscillation of amplitude, a, and frequency, ω. The fluid motion is described by a coordinate frame of reference Oxy centered on the vertical middle line of the body. The calm water