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Velocity and shear stress profiles for tidal effected channels

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ABSTRACT

Tidal current is a periodic movement of unsteady and non-uniform, which owns obvious acceleration and deceleration process. The vertical distributions of velocity and shear stress in tidal flow possess the extraordinary features from the unidirectional flow. From the fundamental tidal movement equation, a simple-structure velocity profile formula is deduced which is capable of describing the velocity states from bypass to fully-developed, and it can express the velocity distribution of which the maximum velocity is not near the water surface. Based on the velocity profile, the tidal shear stress profile expression is derived and it can depict the characteristics commendably that the shear stress obeys the concave distribution within accelerating flow and a convex distribution within decelerating flow. Meanwhile, the computed values of the surface shear stress, on the conditions of acceleration or deceleration, conform to the fact the surface shear stress is not always equal to zero. Following the results of field observations, indoor experiments and numerical models, the bed shear stress is rather a variable dominated by tidal phase than a constant during the tide. The bed shear stress formula in the tide is derived with the definition raised by Soulsby, which is verified with the measured materials. The results are good agreeing with the field data.

1. Introduction

In estuaries water areas and tidal effected channels, a large amount of hydraulic properties all appear different characteristics from the unidirectional flow. The vertical distribution of the velocity in tidal current comes into being periodical variation, and the maximum velocity do not turn up at or near the water surface steadily. The cyclic changing of surface gradient leads to the velocity accelerating or decelerating alternatively, bringing about the non-linear shear stress profiles for tidal effected channels and periodically variable shear stress including surface and bed sections. As vital dynamic characteristics of the turbulent flow, the velocity and the turbulent shear stress, as well as the vertical distribution of which, are closely related to water resistance, suspended sediment transport and suspended load profile(Ni et al., 2012). The velocity profile, as the most essential feature of the stream flow, decides plenty of the water properties, and different velocity profiles correspond to different velocity gradient distributions, which might impact suspended sand profiles directly. The turbulent shear stress would affect the inception of sediment particles, the movement forms of bed load, and the erosion or deposition morphology of bottom bed directly; meanwhile the exchange intensity between suspended load and bed load is influenced significantly. In order to express the vertical distributions of velocity and shear stress for tidal effected water areas objectively, such as estuaries, coastal waters, etc, a number of scholars undertook a series of research, furthermore acquiring numerous results that had been applied to practical engineering. Soulsby (1997) proposed a tidal current velocity throughout the water column empirical formula verified with a great many field data. Nonetheless this formula possesses non-differentiable status at the midpoint of the water depth and the velocity gradient belongs to monotone increasing function. The calculated bypass flow rates by Soulsby (1997) would cause larger deviation for the currents. Li and Feng (2012) presented the velocity profile of sediment-laden flow used for Zhou Shan sea area with the method of sensitivity analysis, yet the profile equation is exclusive of the situation that the maximum velocity do not occur at the water surface or nearby. Li (1992) adopted the boundary condition put forward by Dou G.R. and solved the tidal current turbulent equations to obtain relevant consequence, which had been verified with measured materials. However, the structure of profile formula is too complex to utilize for engineering application and the surface shear stress calculated by this equation is equal to zero, which might not meet the measured. Soulsby and Dyer (1981) put forward the concept of acceleration length and utilized it to modify the logarithm velocity profile obtaining the log-linear formula for expressing the flow states under an acceleration process. But this method is lack of

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physical significance, and this expression would generate larger error when the flow is at bypass moment. Song and Ni (2009) divided the study results about the turbulent shear stress into three categories as follow: For a steady and uniform flow, the vertical distribution of shear stress is linear above a certain distance from the bed surface; For a steady and non-uniform flow, the vertical of shear stress is nonlinear and Lu et al. (2005) gained the relevant distribution with the data of measurement.; For a unsteady and non-uniform flow, the research achievements from Kironoto and Graf (1995) indicated that the shear stress is a concave distribution with acceleration and a convex distribution with deceleration flow rather than a linear distribution. Afzalimenhr and Anctil (2000) confirmed this phenomenon through the experiment. The tidal flow belongs to the unsteady and non-uniform flow owing to the periodic feature and the significant nonlinearity. Song and Ni (2009) expanded the shear stress with MacLaurin series, and constructed a simple formal parabolic distribution based on the related boundary conditions. But the expression of Song and Ni (2009) belongs to an implicit formula which might be difficult to utilize in the practical computation and is lack of considering the influence of the surface elevations and the phase differences between surface elevations and flow rates. Ni et al. (2012) applied distinct boundary condition as Song and Ni (2009) and gained a cubic shear stress profile. Whereas these computed values of surface shear stress are all equal to zero on account of the choice of the forms of structure equations, and the equation of Ni et al. (2012) owned approximate defects as Song and Ni (2009). The objective of this study is to gain explicit formulas of velocity profile and shear stress profile which allows conveniently theoretical calculation in other similar hydrodynamic condition. The bed shear stress expression under the tide is provided, and the phase differences would be taken into account in the corresponding derivations and expressions.

2. Governing equations for tidal effected channel

Supposing that the tidal water is homogenous and incompressible, that the turbulent shear stress satisfies the Boussinesq hypothesis, and the advection and horizontal diffusion terms are ignored, the governing equations of tidal effected channel can be simplified as (Fang and Ichiye, 1983)

$$\begin{aligned} \frac{\partial u}{\partial t} &= fv - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial \tau_x}{\partial z} \\ \frac{\partial v}{\partial t} &= -fu - \frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial \tau_y}{\partial z} \end{aligned} \tag{1}$$

where x,y,z are the right handed Cartesian coordinates with the z-axis vertically upwards; t is time; u and v are x and y direction components of the horizontal velocity; P is pressure; ρ is density of tidal water; f is the Coriolis parameter; τ_x and τ_y are x and y direction components of the horizontal shear stress. A projection on X coordinate is made for the convenience of theoretical derivations, which is the direction of tidal currents (Song and Yan, 2006; Swift et al., 1979), Eq. (1) could be written as

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{\partial \tau_X}{\partial z} \tag{2}$$

where $\tau_X = (u\tau_x + v\tau_y)/U$ and $U = \sqrt{u^2+v^2}$ are the whole shear stress and velocity of tidal current along the movement direction respectively. To keep simple, τ is adopted to replace τ_X, meanwhile u is adopted to replace U. Based on the definition of Newton fluid turbulent shear stress, τ could be expressed as $\tau = \rho\varepsilon\partial u/\partial z$, and ε is turbulent momentum transfer coefficient. Due to the relationship $\frac{1}{\rho}\frac{\partial P}{\partial X} = -g\frac{\partial \vartheta}{\partial X}$ under tidal flow, where ϑ is surface water elevation and g is the gravity constant (Song and Yan, 2006), the pressure term may be rearranged thus

$$\frac{1}{\rho}\frac{\partial P}{\partial X} = -gI = -gI_0 \cos(\sigma t + \varsigma) \tag{3}$$

where I is surface gradient, which is changing periodically in tidal flow; I_0 is the amplitude of surface gradient; σ is angular frequency; ς is phase difference between surface gradient and tidal velocity. By uniting the Eq. (2) with Eq. (3), the governing equation of tidal turbulent motion can be obtained as

$$\frac{\partial u}{\partial t} = gI + \frac{\partial}{\partial z}\left(\varepsilon\frac{\partial u}{\partial z}\right) \tag{4}$$

Agnew, 1961 deemed that the position variable acceleration is much less than constant acceleration, and reduced the N–S equations to acquire the same result as Eq. (4) with the method of dimensional analysis when neglecting the effect of mixing of salt-fresh water.

3. Vertical distribution of velocity

In order to solve the governing Eq. (4), the non-slip condition of the bottom and zero surface velocity gradient, which are usually adopted, would be the boundary conditions based on tidal theoretical research, and the power-law velocity profile, commonly applied in steady and uniform flow, is assumed as the initial condition. Namely,

At the bottom, the velocity equals to zero

$$u = 0 \quad z = 0 \tag{5}$$

At the surface, the velocity gradient equals to zero

$$\frac{\partial u}{\partial z} = 0 \quad z = h = h_0 + \eta_0 \cos(\sigma t + \zeta) \tag{6}$$

where, h is the total water depth, η_0 is the amplitude of the surface elevation, ζ is the phase difference between surface elevation and tidal velocity, and h_0 is the still water depth.

At the initial moment, the flow regime could be considered the steady flow and the different vertical velocity-lag should be included as

$$u = u_1\left(\frac{z}{h_0 + \eta_0 \cos \zeta}\right)^m \cos\left(\varsigma + \theta\left(\frac{z}{h_0 + \eta_0 \cos \zeta}\right)\right) \quad t = 0 \tag{7}$$

where u_1 is the surface velocity when $t = 0$, m is velocity index, and its value range is 0.1–0.2. $\theta[z/(h_0 + \eta_0 \cos \zeta)]$ is the current phase-lag function, which shows mainly the phenomena that the velocity near the bottom arriving at the maximum value is faster than the one near water surface because of the different turbulent effects from near bottom to surface of water column. According to the measured data of the Huangpu River (Li, 1973), as Fig. 1 shows, the tidal phase-log may be nearly linear distribution in vertical, yielding $\theta[z/(h_0 + \eta_0 \cos \zeta)] = az/(h_0 + \eta_0 \cos \zeta) + b$, where a and b are undetermined coefficients, which can be calibrated with the observed data. In Fig. 1, the black symbols represent the phase-lag values from the observation, and the solid line stands for the calculated results from phase-lag function. The correlation coefficient between them could reach 0.9795.

The study on momentum transfer coefficient in tidal flow, under the limitation of unidirectional flow or experience, may be relatively less. By reference to the ordinary study outcome, the momentum transfer coefficient can be written temporarily as

$$\varepsilon = \alpha_1 h_0 \overline{u} \tag{8}$$

#where, α_1 is undetermined coefficient, $\overline{u}$ is vertical averaged velocity. To simplify the complexity in solving the governing equation, supposed the momentum transfer coefficient was constant, Eq. (4)