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Joint distributions of wave steepness in narrow band sea states

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ABSTRACT

The joint probability densities of up-crossing wave steepness and height and of up-crossing wave steepness and period are derived from Longuet-Higgins' joint probability density of wave period and amplitude for narrow band spectra. The formulae of the derived bivariate densities are fitted to numerically simulated data with narrow band spectra. Comparisons with bivariate steepness-period and steepness-height densities for ocean and tank waves are also presented. Concordance between joint distributions for different data types is verified with bivariate Kolmogorv–Smirnov test. The marginal probability density of wave steepness is obtained by numerical integration of the joint distributions and is compared with different data.

1. Introduction

Ocean wave steepness is investigated most often because of the danger a steep wave constitutes for ships and floating structures (Kjeldsen, 1997; Guedes Soares et al., 2007, 2008). Costs of damage caused by a wave of large height and steepness make of wave steepness a parameter of great importance. Deep water waves are most often assumed to be a linear superposition of uncorrelated harmonic components and as such they should follow linear wave theory, where water surface elevation has Gaussian distribution and the amplitude distribution is Rayleigh one. Although real sea states differ from this model, especially in case of storms, (Guedes Soares et al., 2004, 2011) linear theory is still most often used for ship and ocean engineering applications due to its simplicity and acceptable error in most of cases.

To describe steepness probabilities often parametric models are developed from ocean wave data. The models use classical steepness definition as well as new steepness parameters concerning full wave or its parts. Myrhaug and Kjeldsen (1984) developed a parametric model of joint probability distribution of vertical asymmetry factor and wave height, as well as a model of joint probability distribution of wave steepness and height. These models however are valid only for time series recorded in storms with extreme waves on the Norwegian continental shelf. Another model presented in Myrhaug and Kjeldsen (1987) describes the joint distribution of crest front steepness and wave height. The probability of occurrence of waves with different steepness is estimated with each parametric model for a family of JONSWAP spectra.

Myrhaug and Kvaalsvold (1995) compared the Myrhaug and Kjeldsen (1984) parametric model of joint pdf of individual wave steepness and wave height with the transformed wave height and wave period pdf's of Cavanié et al. (1976) and Longuet-Higgins (1983) and the Myrhaug and Kjeldsen (1984) field data.

Tayfun (2006) developed an approximate theoretical form of wave steepness distribution for narrow band long crested waves and validated it on the basis of few hour long storm records.

However, histograms of individual wave steepness probabilities in relatively short duration full scale sea states are rarely conclusive enough to be fitted with a theoretical distribution and this probably explains the lack of published models on this parameter. The same difficulties occur when one tries to model probability distributions of maximal local steepness or equivalent to it maximal local velocities of individual wave as shown in Bitner-Gregersen et al. (1995) and Guedes Soares et al. (2007).

Antão and Guedes Soares (2014) presented an approach to approximate the bivariate probability density of steepness and height for one sea state using copulas. That study used tank records with one wave system, what simplified fitting thanks to larger length of records and the fact that one wave system series more often give empirical densities with one peak structure. Marginal densities were fitted with 3-parameter gamma pdf and this allowed obtaining better marginal fits and focusing on modelling of dependence relation between wave steepness and its height with several copula functions.

The main objective of this work is to study the probability distribution of the steepness of individual waves in a sea state. The analysis is based on linear and 2nd order wave data simulated

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numerically for narrow spectra. Bivariate probability density of wave amplitude and period proposed by Longuet-Higgins (1983) is used to derive the joint density of wave steepness and period and the joint density of steepness and height. The derived distributions are compared with the empirical distributions for numerically simulated waves and measured ocean waves.

2. Theoretical background

A theoretical probability density for joint distribution of wave periods and amplitudes is derived by Longuet-Higgins (1983) for narrow-band spectrum and waves defined with their up-crossings of zero level. From his formula the probability density function of wave height and period can be obtained:

$$f(H,T)=L(\nu)\frac{1}{8\sqrt{2\pi}\nu}\left(\frac{H}{T}\right)^2\exp\left\{-\frac{H^2}{8}\left[1+\frac{1}{\nu^2}\left(1-\frac{1}{T}\right)^2\right]\right\} \quad (1)$$

where $L(\nu)$ is a normalization factor:

$$\frac{1}{L(\nu)}=\int_0^\infty\int_0^\infty\frac{1}{8\sqrt{2\pi}\nu}\left(\frac{H}{T}\right)^2\exp\left\{-\frac{H^2}{8}\left[1+\frac{1}{\nu^2}\left(1-\frac{1}{T}\right)^2\right]\right\}dH\,dT \quad (2)$$

Using the expression for deep water steepness:

$$S=\frac{2\pi H}{gT^2} \quad (3)$$

it is possible to obtain the wave period as a function of height and steepness as well as the height as a function of steepness and period, and substitute them in Eq. (1) to get Eqs. (4) and (5) for the joint distribution of steepness and height and joint distribution of steepness and period. The change of variables requires also multiplication of each distribution by a respective Jacobian:

$$p(S,H)=f\left(\sqrt{\frac{2\pi}{g}\frac{H}{S}},H\right)\cdot\left|\frac{D(H,T)}{D(S,H)}\right| \quad (4)$$

where $|D(H,T)/D(S,H)|=(1/2)\sqrt{2\pi H/gS^3}$

$$p(S,T)=f\left(\frac{gTS^2}{2\pi},T\right)\cdot\left|\frac{D(H,T)}{D(S,T)}\right| \quad (5)$$

where $|D(H,T)/D(S,T)|=(gT^2/2\pi)$

After simplification of Eqs. (4) and (5) the formulae for the respective probability density functions are:

$$p(S,H)=L_1\frac{1}{32\pi\nu}\sqrt{\frac{gH^3}{S}}\exp\left\{-\frac{H^2}{8}\left[1+\frac{1}{\nu^2}\left(1-\sqrt{\frac{gS}{2\pi H}}\right)^2\right]\right\} \quad (6)$$

$$p(S,T)=L_2\frac{g^3}{64\sqrt{\pi}\nu^{\frac{7}{2}}}T^4S^2\exp\left\{-\frac{g^2T^4S^2}{32\pi^2}\left[1+\frac{1}{\nu^2}\left(1-\frac{1}{T}\right)^2\right]\right\} \quad (7)$$

The normalization factors L_1 and L_2 are added here to ensure that the probability density functions integrated over the domain of variation of their variables are unity.

Theoretically by integration of Eq. (6) with respect to height, or Eq. (7) with respect to period, it should be possible to obtain the probability density function of wave steepness (see Eqs. (8) and (9)) and both equations should give the same result.

$$p(S)=\int_0^\infty p(S,H)dH \quad (8)$$

$$p(S)=\int_0^\infty p(S,T)dT \quad (9)$$

Both of these equations give integrals of type $d\int_0^\infty x^4\exp\{-ax^4+bx^3-cx^2\}dx$, which has no analytical solution. Nevertheless the integrals in Eqs. (8) and (9) can be evaluated numerically as shown later in this paper.

3. Data description

The fit of the theoretical distributions is performed mainly on numerically simulated data but there are also comparisons with data from ocean and offshore model testing basins. The numerically simulated data are generated from the random phase model on the basis of JONSWAP spectrum. The simulated series are of one peak and narrow spectrum where the spectral width parameter:

$$\upsilon=\sqrt{\frac{m_0m_2}{m_1^2}-1} \quad (10)$$

satisfies the requirement for narrow-band spectrum, that is $\upsilon^2\le 0.36$. Among simulated sea states – 88 of linear and 46 of second order ones – some have parameters (significant wave height, spectral peak period and spectral width parameter) comparable with parameters of ocean sea states and tank series. Additionally the sampling interval and length of the records of simulated series are equal to the DHI tank series.

A method of simulation of nonlinear random seas was sugested by Hudspeth (1975). It involves inverting of the fast Fourier transform algorithm from frequency to time domain. Second order corrections are made in the frequency domain. The nonlinear components appear at frequencies that are sums or differences of linear frequencies. Thus the nonlinear random sea surface is derived from a linear simulation.

The data used in this study were generated and recorded in the offshore model testing basin of the Danish Hydraulic Institute in Hørsholm (DHI). The records contain deep water waves sampled in intervals of $\Delta t=0.2165$ s. In the majority of files there are $N=24{,}001$ ordinates per record, what gives 86 min long time series. A more precise description of sea states is given in Table 1 which shows designed significant wave height and spectral peak period. The real, that is calculated from data, values of the generated sea states can differ a little from the designed ones.

Second set of data generated in a tank comes from Marintek in The Norwegian Marine Technology Research Institute. It was

Table 1
Description of sea states from DHI offshore model testing basin.

No test	H_s [m]	T_p [s]	ν
1	3.6	7	0.26
2	3.6	10	0.31
3	3.6	14	0.36
4	3.6	20	0.40
5	4.6	7	0.28
6	4.6	14	0.36
7	4.6	20	0.42
8	2.3	7	0.23
9	2.3	14	0.40
10	2.3	20	0.50
11	3.6	7	0.20
12	3.6	14	0.31
13	4.6	14	0.29
14	2.3	14	0.36
105	5	10	0.31
106	5	10	0.32
107	5	12	0.35
108	10	12	0.37