Ocean Engineering 99 (2015) 23–33

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Variational solution for the effect of vertical load on the lateral response of offshore piles

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ARTICLE INFO

Article history:
Received 15 April 2014
Accepted 3 March 2015
Available online 24 March 2015

Keywords:
Vertical load
Lateral load
Offshore piles
Variational method
Parameters analysis

ABSTRACT

Pile foundations supporting offshore and coastal structures are usually simultaneously subjected to vertical loading from the superstructures and lateral loading due to wind or wave actions. An analytical model is presented to investigate the effects of vertical loads on the lateral responses of piles applied in such cases. In this model, the response of the soil is given by the fundamental Mindlin's solution for half-space subjected to both concentrated horizontal and vertical loads. The deformations and reaction pressures of the pile are represented by finite series. The responses of the pile are determined by using the principle of minimum potential energy. The proposed model is validated by comparison with the results of field load tests and laboratory load tests. The influence of related parameters including lateral load level, ratio of the vertical to the horizontal loads, pile slenderness ratio and pile flexibility factor has also been studied in this paper.

1. Introduction

Pile foundations supporting the offshore and coastal structures such as harbor constructions and long-span bridges are usually simultaneously subjected to vertical loading from the supported superstructures and lateral loading due to wind or wave actions. Numerous theoretical methods have been developed to analyze the response of such foundations, including the elastic continuum method (Butterfield and Banerjee, 1971; Poulos and Davis, 1980; Chen and Chen, 2008), the theoretical load–transfer curve method (Randolph and Wroth, 1978), and the nonlinear subgrade reaction method (Matlock, 1970; Reese and Welch, 1975; Georgiadis and Butterfield, 1982). In all these models, either the vertical loads or the horizontal loads are considered independently. However, the effect of the vertical loads on the lateral responses of piles is critical as shown in a number of experimental studies. For example, Anagnostopoulos and Georgiadis (1993) investigated the interaction among the axial and the lateral responses of piles in clay with model tests. Zhang et al. (2002) investigated the effects of the vertical load on the group lateral resistances by centrifuge model test. They pointed out that the influence of the vertical loads is closely linked to the pile–soil system. Knappett and Madabhushi (2009) observed the amplifications of the lateral displacements and the unstable collapse in pile groups under the action of significant axial load and in liquefiable soils.

Additionally, by means of a three-dimensional numerical model based on the finite element program system ABAQUS (version 6.8), the interaction effects of combined loading for piles and their dependence on system parameters were further investigated numerically by Achmus and Thieken (2010a, 2010b) and Achmus et al. (2009). Karthigeyan et al. (2006, 2007) also showed the significant influence of vertical loads on pile's lateral response through a series of three-dimensional finite element analyses (GEOFEM3D) on single pile. Altogether, most of the existing numerical investigations suggest a decrease in lateral deflection due to the presence of the vertical loads.

By using local elasto-plasticity yield surfaces that allows coupling of the differential equations for axial and lateral loading, Levy et al. (2005) presented an analytical model that predicts the behavior of a single vertical pile under combined axial and lateral loading. A reduction was observed in the horizontal load required to displace the pile head for inclined loading. Recently, the authors (Liang et al., 2012) proposed an integral equation method to examine the effects of vertical loads on the lateral response of piles. The model shows that the bending moment and the horizontal displacement distribution along the pile increase considerably with the axial load. Altogether, the results given by numerical analyses and analytical investigations are somewhat inconsistent with respect to the effects of vertical loads on the lateral response of piles (Hussien et al., 2014).

This study presents a variational approach to analyze the effects of vertical loads on the lateral responses of offshore piles and to find the reason for the disagreement existing between the analytical and numerical solution. This variational approach was used by Shen

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http://dx.doi.org/10.1016/j.oceaneng.2015.03.004
0029-8018/

et al. (1997) to study the vertical deformation of pile groups in soil. In their study, the soil was modeled by the theoretical load–transfer curves, and the vertical deformation of the pile was represented by a finite series. This variational method was then extended to vertically loaded pile groups in an elastic half-space (Shen et al., 1999), and the laterally loaded piles (Shen and Teh, 2002). In Shen and Teh's (2002) study, two finite series were used to approximate the lateral displacements of piles subjected to a horizontal load and a bending moment at the pile head. More recently, a numerical solution for laterally loaded piles in a two-layer soil profile was proposed by Yang and Liang (2006).

In the present study, the soil is modeled as an elastic half-space, whose response is given by Mindlin's solution. Finite series are used to approximate the horizontal and vertical displacements, the reaction pressures acting on the pile, and the shear stresses of the pile. The principle of minimum potential energy is then triggered to establish the governing equations of the structure. The details of the parameters study, the verification of the proposed model against some field load tests, and the laboratory load tests are discussed.

2. Method of analysis

2.1. Definition of the problem

Consider a pile subjected to both the lateral and the vertical loads, as shown in Fig. 1. In this figure, l and d are the length and the diameter of the pile, respectively; H, M and P are the horizontal load, the bending moment, and the vertical load acting at the head of the pile, respectively; p_z and τ_z are the horizontal reaction pressures and the vertical shear stresses at depth z, respectively; σ_b is the normal stress at the pile base. In this study, the pile is assumed to be embedded in a soil modelled as an elastic half-space. The fundamental Mindlin's solution (1936) for a concentrated horizontal load and a vertical load is used to simulate the response of the soil.

2.2. Potential energy of the pile

The total potential energy π_p of the single pile subjected to both the lateral and vertical loads, as depicted in Fig. 1, can be written as

$$\pi_p = \frac{1}{2}\iiint_V E_p\left(\frac{\partial w_z}{\partial z}\right)^2 \mathrm{d}v + \frac{1}{2}\int_l E_P I_P\left(\frac{\partial^2 \rho_z}{\partial z^2}\right)^2 \mathrm{d}z + \frac{1}{2}\iint_S \tau_z w_z \mathrm{d}s$$

$$+\frac{1}{2}\iint_A \sigma_b w_b \mathrm{d}A + \frac{1}{2}\int_l p_z \rho_z \mathrm{d}z - P_t w_t - H_t \rho_t - \frac{\partial \rho_t}{\partial z} M_t \quad (1)$$

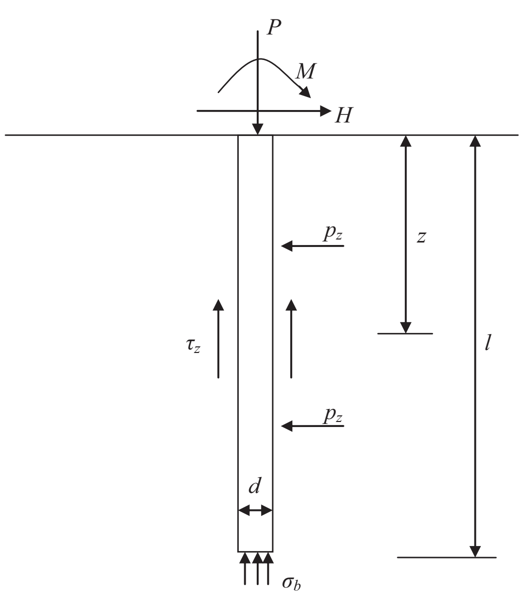

Fig. 1. Forces acting on the pile.

In this equation, the first two terms on the right hand side give the elastic strain energy of the pile, where V is the volume of the pile, E_p and I_p are Young's modulus, and moment of inertia of the pile, respectively, w_z and ρ_z are the vertical and the horizontal displacements of the pile at depth z, respectively. The third and fourth terms on the right hand side of Eq. (1) correspond to the work done by the vertical shear stress τ_z along the pile shaft and the normal stress σ_b at the pile base, respectively, where S, and A are the surface and cross-sectional area of the pile, respectively, and w_b is the vertical displacement at the pile base. The fifth term is the work done by the horizontal reaction pressures p_z along the pile shaft. The last three terms are the works done by the vertical load P_t, by the horizontal load H_t, and by the applied moment M_t at the pile head, where w_t and ρ_t are the vertical and horizontal displacements of the pile at the pile head, respectively.

By using Gauss integration and assuming uniform σ_b and w_b at the pile base, Eq. (1) can be rewritten as

$$\pi_p = U_p + \frac{1}{2}\{P_g\}^T\{w_g\} + \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 \sigma_b w_b + \frac{1}{2}\{P_q\}^T\{\rho_q\} - P_t w_t - H_t \rho_t - \frac{\partial \rho_t}{\partial z} M_t \quad (2)$$

where $U_p = \frac{1}{2}\iiint_V E_p\left(\frac{\partial w_z}{\partial z}\right)^2 \mathrm{d}v + \frac{1}{2}\int_l E_P I_P\left(\frac{\partial^2 \rho_z}{\partial z^2}\right)^2 \mathrm{d}z$. $\{P_g\} = \{P_{g1}, P_{g2}, \cdots, P_{gng}\}^T$, and $\{w_g\} = \{w_{g1}, w_{g2}, \cdots, w_{gng}\}^T$ are the vertical forces and settlement of the pile at the Gauss points, respectively. $\{P_q\} = \{P_{q1}, P_{q2}, \cdots, P_{qnq}\}^T$ and $\{\rho_q\} = \{\rho_{q1}, \rho_{q2}, \cdots, \rho_{qnq}\}^T$ are the horizontal forces and displacements of the pile at the Gauss points, respectively. Here ng and nq are the number of Gauss points chosen for the pile along its shaft. The coefficients in the vectors $\{P_g\}$ and $\{P_q\}$ are given by

$$P_{gi} = \frac{1}{2}\pi d l \eta_i \tau_i \quad (i = 1, 2, \ldots, ng) \quad (3)$$

$$P_{qi} = \frac{1}{2} l d \eta_i p_i \quad (i = 1, 2, \ldots, nq) \quad (4)$$

where η_i is the weighting coefficient of the Gauss points, τ_i is the vertical shear stress at the Gauss points, and p_i is the horizontal reaction pressure at the Gauss points. Eq. (2) can be further expressed as

$$\pi_p = U_p + \frac{1}{2}\{P\}^T\{\rho w\} - P_t w_t - H_t \rho_t - \frac{\partial \rho_t}{\partial z} M_t \quad (5)$$

where

$$\{P\} = \{P_{q1}, P_{q2}, \ldots, P_{qnq}, P_{g1}, P_{g2}, \ldots, P_{gng}, P_b\}^T, \quad P_b = \pi\left(\frac{d}{2}\right)^2 \sigma_b,$$

$$\{\rho w\} = \{\rho_{q1}, \rho_{q2}, \ldots, \rho_{qnq}, w_{g1}, w_{g2}, \ldots, w_{gng}, w_b\}^T$$

For a pile in an elastic half-space, we have

$$\{P\} = [k_s]\{\rho w\} \quad (6)$$

where $[k_s]$ is the soil stiffness matrix and is described in a subsequent section of the paper. Substituting Eq. (6) into Eq. (5) yields

$$\pi_p = U_p + \frac{1}{2}\{\rho w\}^T [k_s]\{\rho w\} - P_t w_t - H_t \rho_t - \frac{\partial \rho_t}{\partial z} M_t \quad (7)$$

2.3. Displacement series of the pile

Three finite series are used to approximate the deformation of the pile under the applied vertical loads, lateral loads, and bending moments. Denoting the horizontal deflection of a pile, ρ_z due to