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Narrow ship wakes and wave drag for planing hulls

M. Rabaud *, F. Moisy

Laboratory FAST, Université Paris-Sud, CNRS. Bât. 502, Campus universitaire, 91405 Orsay, France

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ABSTRACT

The angle formed by ship wakes is usually found close to the value predicted by Kelvin, $\alpha = 19.47^\circ$. However we recently showed that the angle of maximum wave amplitude can be significantly smaller at large Froude number. We show how the finite range of wavenumbers excited by the ship explains the observed decrease of the wake angle as $1/\mathrm{Fr}$ for $\mathrm{Fr} > 0.5$, where $\mathrm{Fr} = U/\sqrt{gL}$ is the Froude number based on the hull length L. At such large Froude numbers, sailing boats are in the planing regime, and a decrease of the wave drag is observed. We discuss in this paper the possible connection between the decrease of the wake angle and the decrease of the wave drag at large Froude number.

1. Introduction

A ship moving on calm water generates gravity waves presenting a characteristic V-shaped pattern. Lord Kelvin in 1887 was the first to explain this phenomenon and to show that the wedge angle is constant, independent of the boat velocity (see, e.g., Darrigol, 2005). According to this classical analysis, only the wavelength and the amplitude of the waves change with the velocity, and the half-angle of the wedge remains to be equal to 19.47°.

In contrast to this result described in many textbooks, we have shown recently that the apparent wake angle α, i.e. the angle of maximum wave amplitude, is not the Kelvin angle at large velocity, but rather decreases as $1/\mathrm{Fr}$, where $\mathrm{Fr} = U/\sqrt{gL}$ is the hull Froude number, based on the boat velocity U and on the waterline length L (Rabaud and Moisy, 2013). We have shown how this decrease can be simply modeled by considering the finite length of the boat. This scaling law $\alpha \propto 1/\mathrm{Fr}$ has recently received an analytical confirmation by Darmon et al. (2014).

Some years before Kelvin's work, William Froude, by towing model boats, observed that at intermediate velocity the hydrodynamic drag increases rapidly with the hull Froude number (Darrigol, 2005). Since then, the computation of hydrodynamic drag has received considerable interest (Michell, 1898; Tuck, 1989; Havelock, 1919), and still represents a challenge for naval architects. The wave drag (or wave-making resistance) R_W is the part of the hydrodynamic drag that corresponds to the energy radiated by the waves generated by the hull translation. For a displacement hull sailing at large velocity (Froude number in the range 0.2–0.5) the major part of the hydrodynamic drag is actually due to the wave drag.

In this paper we review some recent results about the Froude number dependence of the wake angle and the wave drag, and discuss the possible link between the decrease of these two quantities for planing sailing boats at large Froude number.

2. Wave pattern

When a boat sails on calm water at constant velocity U, the waves present around and behind the hull are only those that are stationary in the frame of reference of the boat. For a given wave of wavenumber k propagating in the direction θ with respect to the boat course (Fig. 1), this stationary condition gives

$$U \cos\theta(k) = c_\varphi(k) \tag{1}$$

where $c_\varphi(k)$ is the phase velocity of the wave.

Because of the dispersive nature of gravity waves, c_φ is a function of the wave number, $c_\varphi = \sqrt{g/k}$, implying that for a given propagation direction θ only one wavenumber is selected by Eq. (1):

$$k(\theta) = \frac{g}{U^2 \cos^2\theta}. \tag{2}$$

As a consequence, the smallest wave number (i.e. the largest wavelength) compatible with the stationary condition is $k_g = g/U^2$, and corresponds to waves propagating in the boat direction ($\theta = 0$). These so-called transverse waves are visible along the hull and following the boat.

Importantly, energy propagates at the group velocity and not at the phase velocity, and for gravity waves the group velocity is equal to half the phase velocity ($\mathbf{c_g} = \frac{1}{2}\mathbf{c}_\varphi$) (e.g., Lighthill, 1978).

* Corresponding author.
E-mail addresses: marc.rabaud@u-psud.fr (M. Rabaud), frederic.moisy@u-psud.fr (F. Moisy).

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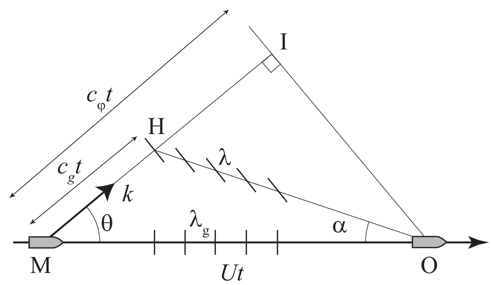

Fig. 1. Geometric construction of the wave pattern and angle definitions for a boat sailing at constant velocity U (Crawford, 1984).

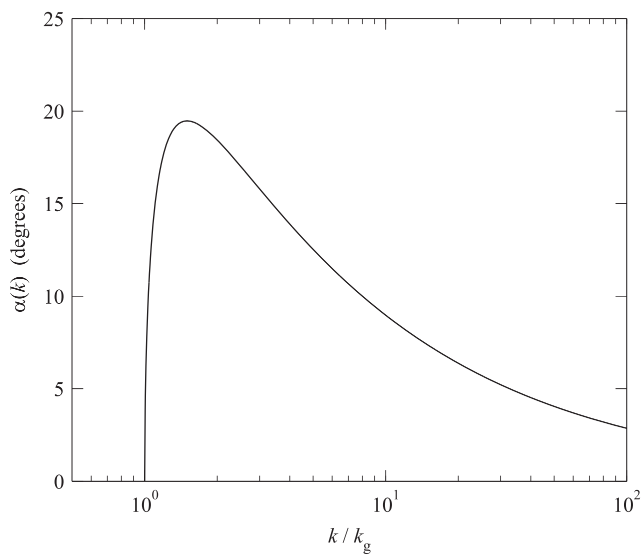

Fig. 2. Radiation angle $\alpha(k)$ as a function of the wavenumber ratio k/k_g (Eq. (3)), where $k_g = g/U^2$.

It follows from this ratio that the *radiation angle* $\alpha(k)$, along which the energy of a wavenumber k propagates in the frame of the disturbance, is given by (Keller, 1970; Rabaud and Moisy, 2013)

$$\alpha(k) = \tan^{-1}\left(\frac{\sqrt{k/k_g - 1}}{2k/k_g - 1}\right). \quad (3)$$

The plot of $\alpha(k)$ (Fig. 2) shows that for any given angle α smaller than 19.47° there are two possible values of k that correspond to two directions θ (Eq. (2)). One solution corresponds to transverse waves (smaller θ) and the other one to divergent waves (larger θ). The angle α takes its maximum value $\alpha_0 = 19.47°$ for $k_0/k_g = 3/2$, and no waves can be observed beyond this angle: this is the well known Kelvin angle, which corresponds to a cusp in the wave pattern. If the disturbance is a point source exciting a broadband spectrum of wavenumbers, then an accumulation of energy must take place at $k_0/k_g = 3/2$ (because the angular energy density $E(\alpha) = E(k)|\partial\alpha/\partial k|^{-1}$ diverges at k_0, with $E(k)$ the spectral energy density radiated by the disturbance), so the cusp is also the locus of maximum amplitude of the waves.

In reality, a boat cannot be described by a single point source: all the points of the hull act as wave sources and the detail of the amplitude of the wave depends on the exact shape, trim, sinkage of the hull, and on the Froude number. For example, for a poorly streamlined hull at low Froude number, two V-shaped wakes are visible, one originating from the bow and the other from the stern.

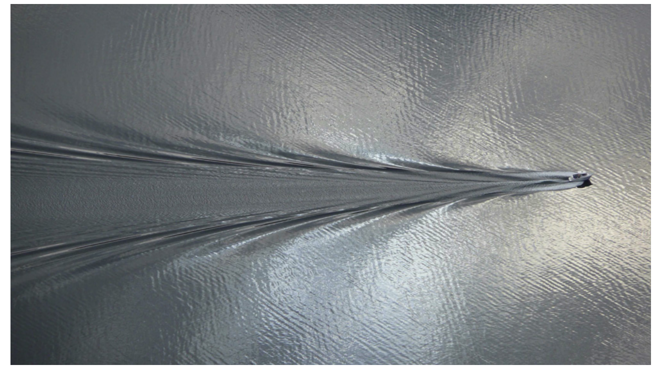

Fig. 3. Photograph of a fast planing motor-boat exhibiting a narrow wake (source: http://en.wikipedia.org/wiki/Wake).

In general the waves generated by a boat are characterized by a spectrum containing one or several characteristic length scales, corresponding to specific ranges of wavenumbers, so the maximum of wave amplitude is not necessarily located at the cusp angle (Lighthill, 1978; Carusotto and Rousseaux, 2013).

3. Wave angle for rapid boats

The commonly accepted result of Kelvin of a constant wake angle of 19.47° is called into question by numerous observations of significantly narrower wakes for planing boats at large velocity. This is illustrated in Fig. 3, showing a wake angle of order of 10°, significantly smaller than the Kelvin prediction.

Analyzing a set of airborne images from Google Earth©, we measured the wake angles and the Froude numbers for boats of various sizes and velocities. Using the scale provided on the images, we measured the overall length of the boat (assumed to be equal to the waterline length L) and the wavelength of the waves on the edge of the wake. From this wavelength the boat velocity U is determined using Eq. (2) and the Froude number is then computed. Our data clearly show a decrease of the wedge angle for Froude numbers larger than 0.5 (Fig. 2 of Rabaud and Moisy, 2013). Values as small as 7° were observed.

Wake angles smaller than the Kelvin prediction can be explained as follows. The key argument is that a moving disturbance of size L cannot excite efficiently the waves significantly smaller or larger than L. This is a general property of dispersive waves, analogous to the Cauchy–Poisson problem for the temporal evolution of an applied initial disturbance of characteristic size L at the free surface of a liquid (Havelock, 1908; Lighthill, 1978): the wave packet emitted by the disturbance travels at the group velocity $c_g = \frac{1}{2}\sqrt{g/k_f}$ corresponding to a wave number k_f of the order of L^{-1}, and the characteristic wavelength at the center of the wave packet is of the order of L. It is therefore possible to model the angle of maximum wave amplitude by simply considering that the energy radiated by the boat is effectively truncated below the wavenumber L^{-1}. At large boat velocity this wavenumber can be larger than the wavenumber $k_0 = 3g/2U^2$ which corresponds to the maximum Kelvin angle. Since only the wavenumbers of order of k_f are of significant amplitude, the angle of maximum wave amplitude is given by Eq. (3) evaluated at $k_f \simeq L^{-1}$. This simple model shows that the apparent wake angle is given by the Kelvin prediction as long as energy is supplied to k_0 (small Fr), but it is a decreasing function of velocity for Fr larger than a crossover Froude number Fr_c. Choosing $k_f = 2\pi/L$ (the exact prefactor depends on the shape of the disturbance spectrum) gives