



On risk attitude and optimal yacht racing tactics



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ABSTRACT

When the future wind direction is uncertain, the tactical decisions of a yacht skipper involve a stochastic routing problem. The objective of this problem is to maximise the probability of reaching the next mark ahead of all the other competitors. This paper describes some numerical experiments that explore the effect of the skipper's risk attitude on their policy when match racing another boat. The tidal current at any location is assumed to be negligible, while the wind direction is modelled by a Markov chain. Boat performance in different wind conditions is defined by the output of a velocity prediction program, and we assume a known speed loss for tacking and gybing. We compare strategies that minimise the average time to sail the leg with those that seek to maximise the probability of winning, and show that by adopting different attitudes to risk when leading or trailing the competitor, a skipper can improve their chances of winning.

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1. Introduction

In this paper we model and analyse the problem faced by a skipper who wants to sail an upwind leg of a yacht race, rounding the mark before his opponent. This problem falls into the category of stochastic shortest-path problems, where the cost function to be minimised is the time needed to reach the mark, and it depends on stochastic quantities such as wind direction. Many problems fall into this category and involve routing for emergency response, both civil (Yamada, 1996) and military (Resch et al., 2003), and applications in logistics (Fleischmann et al., 2004) and transport (Shuxia, 2012). The aim is to find a path between two vertices of a graph such that the sum of its constituent edges, often representing a cost, is minimised. When cost depends on random quantities this becomes a stochastic problem, and the standard objective is to minimise expected costs (where costs include time) (Bertsekas and Tsitsiklis, 1991). For yacht races, models which minimise the expected time to finish, or to reach the next mark, have been studied in a number of papers (Philpott and Mason, 2001; Philpott, 2005). This might be appropriate in fleet races where corrected time over a number of races forms a basis for scoring points. Even so, such scoring systems assign rankings in each race and it is well known that rank-based scoring leads to different incentives than those from performance on average (Anderson, 2012).

As observed in Philpott (2005) rank-based scoring takes its most extreme form in match racing, where the objective is to maximise the probability of arriving before the competing yacht. Indeed the time difference between the two boats is not of interest, as opposed to its sign. In this context, the attitude towards risk of the skipper assumes a greater importance. The aim of this work is to show that by changing the skipper's attitude to risk, it is possible to define a strategy that performs better in match races than strategies aimed at minimising the expected time to finish.

Of course, in most forms of match racing, the interaction between the boats is important. A leading yacht will attempt to cover a trailing yacht, not only for tactical reasons, but also to spill turbulent air on the trailing yacht's sails to reduce their drive. Forcing another boat to tack to avoid a collision is also a tactical ploy to increase a yacht's advantage. In this paper we choose to ignore these effects, as well as assuming identical yachts and crew expertise. This is done for modelling convenience as well as simplicity. By focusing solely on risk attitude we can see to what extent this is important, other effects being equal.

The paper is laid out as follows. In the next section we describe the model of the yacht and basic sailing strategy for the upwind leg of a match race. We then review dynamic programming as an approach to finding the strategy that minimises the expected time to reach the next mark. The following section shows how this is implemented in a routing model that accounts for different risk attitudes of the skipper. We then present the results of some simulations of the strategies that emerge from the routing model.

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1.1. Sailing strategy

The speed of a sailing yacht depends on the wind speed and on the angle between boat heading and wind direction. It is usually expressed as a polar diagram like the one shown in Fig. 1. The numbers around the semicircle represent different true wind angles, while the radial ones represent the boat speed. The red line corresponds to the plot of boat speed for a particular true wind speed. While no direct course is possible straight into the wind, it is possible to sail upwind with an angle between wind direction and sailed course which is usually between 30° and 50° . Sailing closer to the wind direction (lower angle) makes the course shorter, but when sailing at higher angles a boat is faster. Velocity made good (VMG) is the component of yacht velocity in the wind direction. With a constant wind direction from the top mark, an optimal policy maximises VMG. This is typically attained at a true wind angle of around $40\text{--}45^\circ$ (as in this example). In a polar diagram like the one in Fig. 1, it is possible to find the maximum VMG for a given wind speed by finding the intersection between the polar corresponding to the wind speed and the line perpendicular to the upwind direction. For this reason the common route towards an upwind mark, or in general towards the direction from which the wind blows, is a zigzag route. Such a route requires changes of direction which are called *tacks*. When manoeuvring for a tack, a boat points for a few seconds directly into the wind, therefore causing a temporary decrease in boat speed. If the wind is constant during the race and all over the racing area, trying

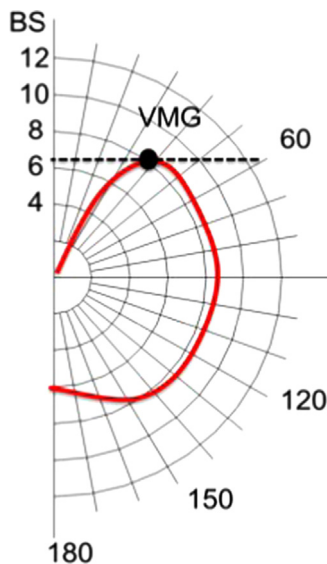


Fig. 1. Example of a polar diagram (velocities in m/s and angles in degrees). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

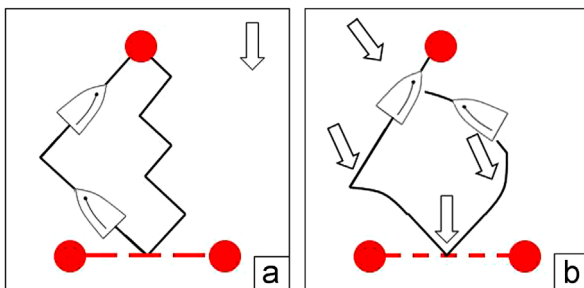


Fig. 2. Example of upwind routes. (a) Constant wind and (b) left wind shift.

to do the minimum number of tacks is the best choice. Fig. 2(a) shows two possible routes. In a constant wind, the route on the left is faster because it involves just one tack. Fig. 2(b) shows a situation in which the wind shifts towards the left over the duration of the leg. The best policy in this case is to go to the left of the course (referred to as being on *starboard tack*), and then tack and point towards the mark, while a myopic policy that begins the race going to the right (referred to as being on *port tack*) turns out to be suboptimal.

In real races the evolution of the wind can be much more complicated than these examples, with temporary shifts or gusts that a sailor seeks to take advantage of. Moreover wind has a random component. While racing, it is difficult to know how the wind is behaving at another location, or to foresee how it will behave once that point is reached. In the presence of randomness the optimal course in Fig. 2(b) might turn out to be worse than a myopic policy that tacks on every wind shift. For this reason sailors tend to try and stay in the centre of the course to enable shifts in wind direction to be exploited by tacking, while avoiding the risk of overlaying the mark.

In the presence of a competitor, a policy that avoids the course boundaries while staying close to the competitor reduces the risk of being beaten, at least when the competitor is the trailing boat. On the other hand, when the competitor is leading, it can make sense for a skipper to take a risk and explore the corners of the course hoping for a favourable wind shift. This is the phenomenon that we seek to model in this paper.

1.2. Dynamic programming

Finding an optimal set of tacks when the wind varies randomly requires a *stochastic dynamic* optimisation model. In contrast to the deterministic case, a solution does not consist of a single optimal path for a specific wind realisation, but a *policy* that is optimal over a range of wind realisations. Policies can be computed a priori and respect the principle of optimality: an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (Bellman, 1957). A policy that respects this principle can be found with *dynamic programming* (Bertsekas, 1995). Dynamic programming has been successfully applied in sailing in both ocean races and short course racing (see Philpott and Mason, 2001; Philpott, 2005). In this work we adapt the short-course model described in Philpott and Mason (2001) and Philpott (2005) with the aim of incorporating the skipper's attitude towards risk in their actions.

The risk that a skipper is willing to take is usually influenced by his position with respect to the opponent. A common behavioural pattern is to be conservative, or *risk averse*, when in a leading position, while being *risk seeking* when losing. Here we interpret risk aversion as being pessimistic about wind shifts, believing that any shifts we observe will not be to our advantage. In contrast, a risk-seeking skipper will be optimistic about wind shifts and act as if these are more likely to be to his advantage. Such attitudes can be modelled by altering the transition probabilities of the process that defines wind shifts.

To understand the effect of risk-averse or risk-seeking skippers, we develop a race modelling program (RMP) for simulating races between two boats. The first RMP was developed in 1987 for the America's Cup syndicate Stars and Stripes and is described in Letcher et al. (1987). Since then, RMPs have been used mainly in America's Cup applications to compare different designs (see e.g. Philpott et al., 2004). In our case, since we are interested in comparing tactical choices, we model two identical boats (i.e. they have the same polar diagram).

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