Ocean Engineering 88 (2014) 46–54

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Extended wave power formulation by perturbation theory and its applications

Abdüsselam Altunkaynak

Department of Civil Engineering, Istanbul Technical University, Maslak, 34469 Istanbul, Turkey

ARTICLE INFO

Article history:
Received 22 July 2013
Received in revised form
7 June 2014
Accepted 13 June 2014
Available online 5 July 2014

Keywords:
Perturbation
Wave power
Weibull probability distribution
Significant wave height
Wave period

ABSTRACT

Water wave is a powerful source of energy; the problem is harnessing and making use of it. Wave power is a renewable, pollution-free and sustainable energy source. Since wave power varies with the wave period and the square of significant wave height, it is necessary to quantify the variations and statistical characteristics of these parameters with time. The aim of this study was to derive a general wave power equation that can be used in order to determine the mean and the standard deviation of the wave parameters using perturbation approach. The result of the study showed that Weibull probability distribution function is able to accurately simulate wave power. The wave power equation developed in this study is a general formula and can be applied at any desired region. The equation was then applied to five stations and it was seen that the perturbation approach yields better statistical values compared to results determined using classical statistical approach. This indicates that it is necessary to use perturbation approach to achieve optimum wave turbine design.

1. Introduction

Consumption of fossil fuel emits carbon dioxide, subsequently causing global warming and climate change. Global warming and climate change have become serious threats to our daily life. In this context, clean energy sources are believed to play a major role in addressing these threats and meeting our future energy needs (Wuebbles and Jain, 2001). Moreover, renewable and clean energy sources have been preferred by countries with high living standard as renewable and clean energy sources are considered to be environmental friendly. Ocean wave power is one of potential clean and renewable energy sources. Studies on renewable energy sources, especially on wave power have gained acceleration in recent years due to economic reasons, fluctuations in the price of petroleum and natural gas, and environmental concerns. In addition, wave energy has a considerable advantage as it can provide energy continuously.

Wave power is highly sensitive to variations in significant wave height and wave period at a particular area. In order to properly assess the efficiency of wave power conversion device, the knowledge about statistical characteristics of the significant wave height and wave period is required. Design considerations also include the transient variability of wave power levels. This is demonstrated in the study by Mackay et al. (2010), which focused on the uncertainty in the estimation of the energy yield from a wave power converter.

Different aspects of ocean wave power have been studied so far by many researchers (Izadparast and Niedzwecki, 2011; Iglesias and Carballo, 2009; El Marjani et al., 2008; Ozger and Sen, 2008; Beyene and Wilson, 2006; Antonio and Falcao, 2004). Ozger et al. (2004a) investigated the statistical behavior of wave power at a single site which can be derived by considering simultaneous variations in the significant wave height and wave period. Further, Ozger et al. (2004b) derived a much generalized wave power formulation by considering temporal variations both in the significant wave height and wave period. Reikard (2009) evaluated the ability of time-series models to predict the energy from ocean waves. In addition, joint probability distribution of individual wave heights and periods was investigated and compared to model distributions by various researchers (Longuet-Higgins, 1975; Cavanier, et al., 1976; Goda, 1978; Satheesh et al., 2005; Zhang et al. 2013). Values of the average significant wave height were used in most of the studies at different sites as mentioned in the literature.

Wave height and wave period statistics such as the mean and the standard deviation, which are related to actual skewed distribution of the significant wave height and wave period, are related to wave power estimations. Obviously, determination of the mean and the standard deviations of the significant wave height and wave period would require many observations and measurements (Khodaparast et al., 2008). Alternatively, a perturbation approach, which is an approach that comprises mathematical methods for finding an approximate solution of a problem

E-mail address: altunkay@itu.edu.tr

http://dx.doi.org/10.1016/j.oceaneng.2014.06.006
0029-8018/

Nomenclature

λ_1	Scale parameter [m] for significant wave height
λ_2	Scale parameter [s] for wave period
k_1	Dimensionless shape parameter for significant wave height
k_2	Dimensionless shape parameter for wave period
$E(.)$	Arithmetical average
Var(.)	Variance
$f(\ )$	Probability density function
n	Order term
P	Wave power [kw/m]
PDF	Probability distribution function
W-PD	Weibull probability distribution
W-PDF	Weibull probability distribution function
H	Significant wave height [m]
$\overline{H}$	Average significant wave height [m]
T	Wave period [s]
$\overline{T}$	Average wave period [s]
H'	Perturbation term of significant wave height [m]
T'	Perturbation term of wave period [s]

Greek letters

Γ	Gamma function
σ_p	Standard deviation
α, β	Dimensionless parameters

that cannot be solved exactly by starting from the exact solution of a related problem, may yield more accurate estimations. Four moments of significant wave height (mean, standard deviation, skewness, and kurtosis) can be characterized and considered in perturbation approach as opposed to the classic approach.

The aim of this study was, therefore, to obtain a general wave power equation by using perturbation approach. There are various types of probability distribution functions. However, Weibull probability distribution (W-PD) is the most commonly used method for identifying the significant wave height distribution (Ochi, 1998, Battjes, 1971; Guedes Soares and Henriques, 1996). In addition, Muraleedharan et al. (2007) used the modified Weibull probability distribution for the significant wave height simulation. They derived parametric relations to estimate various significant wave height statistics including extreme wave heights. In their study Chu et al. (2010) concluded that the probability distribution function of the global significant wave heights approximately satisfies the two-parameter Weibull distribution. Prevosto et al. (2000) indicated that Weibull distribution is a natural choice, particularly for the maximum wave crest and also stated that Weibull distributions for wave height is, therefore, a reasonable choice. Taking these previous works into consideration, Weibull probability distribution function was employed in order to compute the mean and the standard deviation in this study.

2. Methodology

The general expression of wave power (kW/m) given by Tucker and Pitt (2001) (Eq. 1) was used as a base to extend wave power formulation by considering a perturbation theory. Then, the classical wave power equation and the extended wave power equation based on perturbation theory were fitted to Weibull probability distribution. The mean and the standard deviation were generated considering Weibull probability distribution.

The general formulation developed in this study was then applied for five stations. The hourly significant wave height series of the 5 stations used for the application were obtained from NOAA's National Data Buoy Center for the period from January, 1997 through December, 1998 (http://www.ndbc.noaa.gov/climate.phtml). The measurement stations from which the data were obtained are identified as 46002, 46005, 46006, 46011 and 46012. The stations are located in the western coast of United States, Pacific Ocean, and are shown in Fig. 1. Table 1 represents the location and some statistical properties of the time series of stations including the depths of the stations. The flow chart showing the steps followed in the calculation of wave power is depicted in Fig. 2.

3. Development of the general wave power formula

3.1. Extended wave power formulation by perturbation theory

Planning and designing of ocean structures should be undertaken based on significant wave height. The general expression of wave power (kW/m) is given as follows (Tucker and Pitt, 2001):

$$P = 0.49\, T\, H^2 \quad (1)$$

where H is the significant wave height (m) and is taken as the mean of the highest 1/3 of all waves seen in a period of time; T is the wave period (s). The wave period used in this study is the mean wave period (seconds) of all waves during the *20*-min recording period.

The squared significant wave height (H^2) and wave period (T) are considered to be the major variables in Eq. (1). In such a case, the expected value of statistical wave power can be described as follows:

$$E(P) = 0.49 E(T) E(H^2) \quad (2)$$

Here, $E(.)$ is the expected value of P and is equivalent to the mean (arithmetical mean) for a long time series.

Perturbation theory, which is based on the concept that considers the fluctuation around the mean value of a variable, has been studied in the literature especially in turbulent flow in channels by Taylor (1915). In this present study, the significant wave height and wave period records, which present fluctuations around their mean values, are taken as perturbation variables. Hence, one can see that the instantaneous significant wave height (H) and the instantaneous wave period (T) values are composed of two components. These are the mean wave period, $\overline{T}$ and mean significant wave height, $\overline{H}$, and the perturbation terms, T' and H' as shown as follows:

$$T = \overline{T} + T' \quad (3)$$

and

$$H = \overline{H} + H' \quad (4)$$

where $\overline{T}$ and $\overline{H}$ are the arithmetic means and T' and H' are the perturbation terms which are random fluctuations around these means as shown in Fig. 3.

A well-known feature of the perturbation term is that its expected value or the mean value is equal to zero, i.e., $E(T') = 0; E(H') = 0$ and its variance is equal to the variance of the instantaneous significant wave height data, i.e., $Var(T) = E(T'^2); \quad Var(H) = E(H'^2)$. If Eqs. (3) and (4) are substituted into Eq. (1), the following expression is obtained:

$$P = 0.49(\overline{T} + T')(\overline{H} + H')^2 \quad (5)$$