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Optimal control of an array of non-linear wave energy point converters

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ABSTRACT

The paper deals with the optimal feedback control and sub-optimal causal feedback control of an array of wave energy point absorbers using the reactive forces from the power take-off systems on the point absorbers as control forces. The dynamic coupling of the absorbers via the radiation wave forces and control forces are taken into account. Assuming linear wave mechanics the optimal control law is shown to be a non-causal feedback controller with feedback from measurement of the displacement, velocity, and acceleration of all floaters. i.e no wave load estimation or prediction is assumed. The control law will be optimal for any 2D or 3D irregular sea-state, as well as during the transient phase. To circumvent the non-causality problem related to the optimal controller law, a causal closed loop controller is suggested based on a slightly modified optimal control law. The controller contains an undetermined symmetric positive definite gain matrix. Since, the response of the array is narrow-banded at optimal control, this matrix has been chosen as the radiation damping matrix at the peak angular frequency. The causal controller is optimal under monochromatic wave excitation and close to optimal for irregular sea-states.

1. Introduction

A Wave Energy Converter (WEC) may be defined as a dynamic system with one or more degrees of freedom using the reactive forces from an attached hydraulic power take-off systems on the converter as active control forces with the intention to convert a maximum amount of energy in the waves into mechanical energy stored in the oscillating system. Correspondingly, we shall refer to the said reaction forces as control forces in the following. A point absorber is a WEC with a size that is small compared to the dominating wave length. The oscillating point absorbers are attached to a shaft, which drives a generator with a fixed or an inertial support. This produces a low frequency irregular alternating current, which is next transformed into a grid-compliant alternating current. The idea of extracting energy from the waves is very old and many WEC devices have been proposed (Falnes, 2002). This has initiated commercial WEC projects using devices such as different buoy concepts, Oscillating-Water-Column (OWC) plants, the Pelamis (Pelamis Wave Power Ltd., 2012), overtopping WEC types like the Wave Dragon ApS (2005), point absorber approaches used for the Wavestar device, Wave Star A/S (2012), or the SEAREV multi-degree-of-freedom point absorber device, Ruellan et al. (2010). Many control strategies have been indicated and reviewed in French (1979) and Falnes (2007). Mechanical energy is stored in the WEC when the dynamic hydrodynamic force is performing positive work on the WEC during a certain time interval. Obviously, this will always be the case if the dynamic hydrodynamic force and the work-conjugated absorber velocity has the same the sign “are in phase”. Enforcement of this condition by the control force forms a guideline for any control effort of a WEC. Especially, under monochromatic wave excitation this is achieved if the absorber displacement is in resonance with the harmonic varying excitation, e.g. Nielsen (2004). Basically, the active control of the point absorbers may be either of the open-loop (feed forward) or of the closed-loop (feedback) type. Open-loop control implies that the control demand is determined based on observation (measurement) of the wave excitation force. Open-loop does not affect the dynamics of the system, i.e. angular eigenfrequencies and structural damping ratios are unchanged by the control. Closed loop control is entirely based on the observed motion of the absorbers. Typically, this involves the displacement, velocity and acceleration components, which easily can be measured by accelerometer or laser vibrometer measurements onboard the floating devices. A closed loop control always change the dynamic properties of the system (inertia, damping or stiffness parameters), as specified by the poles and zeros of the frequency response functions relating the wave excitation forces to the displacement responses of the absorber system. The simplest

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closed loop control laws are achieved by so-called proportional, derivative and acceleration controllers, where the control force is specified to beproportional to and oppositely directed to the displacement, the velocity and the acceleration of the WEC. The proportional and acceleration control force components change the stiffness and mass of the absorber, and hence the angular eigenfrequency. In contrast, the derivative controller has merely insignificant influence on the angular eigenfrequency. Derivative control and proportional control are denoted as linear damping and reactive control in wave energy applications, see e.g. Hansen et al. (2011). Finally, integral control can be introduced for which the control force component appears as a convolution integral of the absorber velocity with respect to a given impulse response function. It turns out that integral control needs to be introduced, if perfect phase locking between the wave excitation force and the velocity of the absorber is attempted at all frequencies (Nielsen et al., 2013). Due to the coupling via the radiation and control force vectors the power absorbtion of an array of wave energy converters depends on position of the absorbers as well as the direction and frequency of the impending wave train. The performance is measured by the interaction factor, defined as the maximum average power absorbed by the array divided by the number of absorbers in proportion to the maximum power absorbed by a single isolated absorber. The interaction factor can be both larger and smaller than one corresponding to positive and negative interaction of the array. The literature on the interaction factor of wave energy absorbers in irregular sea-states has been reviewed in Babarit and Hals (2011). The effect of array interaction was first studied by Evans (1979), who provided a theoretical solution for the mean power absorption in monochromatic waves assuming that the hydrodynamic modelling coefficients of all elements in the array are known. Antonutti and Hearn (2011) calculated the power of the array in monochromatic and irregular seastates using a derivative control law with a diagonal gain matrix, i.e. no control coupling was assumed. The optimal control gain factors were calculated based on a numerical optimization. Folley and Whittaker (2009) investigated the array effect in monochromatic waves using a sub-optimal control law based on a diagonalization of both the frequency response matrix of the dynamic system, i.e. the couplings via the radiation damping matrix as well the complex control gain matrix were ignored. Folley and Whittaker (2011) also considered the influence of the phase of the individual harmonic wave components on the absorbed power of the array, and concluded that this is to be unimportant, so the modelling of the wave excitation forces may be based merely on the auto-spectral density function of the sea-state without consideration to the phase spectrum. Westphalen et al. (2011) considered the difference between independent isolated control and coupled global array control, using an open-loop type of control law. Guidelines for the optimal array layout was studied by Babarit (2013), recommending smaller arrays with as large a distance as possible between the members. Cruz et al. (2010) provided numerical results for interaction factor as a function of the array layout assuming derivative control with a diagonal control damping matrix. The control of an array of point absorbers with constraint on the allowable displacement and the available power take off force of the absorbers has been considered by Li and Belmont (2013) using model predictive control, assuming linear hydrodynamics. The same problems were studied by Bacelli and Ringwood (2013a,b). The prediction of future velocities and power take off forces of the absorbers was performed based on truncated Fourier series, and the constraints on the displacements and the power take off forces were reformulated in terms of the amplitudes of the harmonic components of the Fourier series. In the present paper the non-constrained control law is at first derived, which optimizes the mean absorbed power for an array of point absorbers of the array of point absorbers, assuming linear hydrodynamics and non-linear buoyancy forces. The optimal control law does not include any unspecified gain matrix or integral kernel to be determined by a succeeding optimization procedure. Instead basic hydrodynamic quantities are entering, such as the added hydrodynamic mass matrix at infinite frequencies and the impulse response matrix for the radiation force vector, which have to be calculated numerically by a linear finite element or boundary element program. The optimal control demand at a given instant of time depends on the future velocities of the point absorbers, which needs to be predicted within the required control horizon. The prediction is related with uncertainty, resulting in a reduced efficiency of the controller. As an alternative a causal sub-optimal control law with an unspecified gain matrix is suggested, obtained by a minor modification of the optimal control law. In a previous paper the authors analyzed the optimal control of a single non-linear wave energy point converter, where the corresponding undetermined gain factor was calibrated based on a stochastic dynamic analysis (Nielsen et al., 2013). The corresponding approach for the multi-absorber case turn out to be somewhat complicated. Instead the said gain matrix is chosen as the radiation damping matrix calculated at the angular peak frequency of the wave excitation. Both the optimal and the sub-optimal causal control strategies take the coupling between the absorbers via the off-diagonal terms in added mass matrix and the impulse response matrix of the radiation force vector into consideration, without increasing the computational effort significantly compared to independent control of the absorbers. In a numerical example it is demonstrated that the sub-optimal causal control is close to optimal as long as the hydrodynamic parameters can be calculated with sufficient accuracy

2. Equation of motion of an array of point absorbers

Fig. 1 shows an array of n separate point absorbers, each described by a single degree of freedom. The position and motion of the absorbers are described in the indicated $(x,y,z)-$coordinate system, where the (x,y)-plane is placed in the mean water level (MWL), and the z-axis is orientated upwards.

Although the equation of motion and the control law will be formulated for a system of n heave absorbers similar to the one shown in Fig. 2, the results may easily be carried over to other systems of single-degree-of-freedom floaters by a slight modifications. The devised control laws apply to any sea-state. However, explicit solutions will only be indicated for two-dimensional (plane) regular or irregular waves, which are propagating in a given direction in the (x,y)-plane defined by the wave number vector $\mathbf{k}$ as shown in Fig. 1. The motion $v_j(t)$ of absorber j is defined as the displacement relative to the static equilibrium state, where the static buoyancy force $f_{bj,0}$ is balancing the gravity force m_{jg} and a possible static pre-stressing force from the power take-off system $f_{pj,0}$. m_j denotes the structural mass including ballast, and

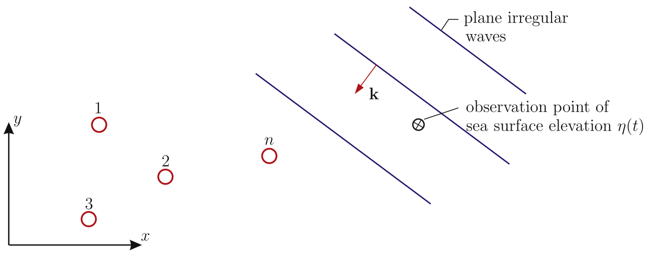

Fig. 1. Array of point absorbers.