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A theoretical solution to dispersion coefficients in wave field

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ABSTRACT

Dispersion of mass concentration due to ocean waves has characteristics different from that induced by river flows. Theoretical formulae of first and second order dispersion coefficients, including both mean and oscillating parts, are derived by successive approximation for flow field composed of tidal current, Stokes drift and first order wave orbital velocity, in which the second order dispersion coefficient is first introduced. The significant contribution of wave orbital velocity is investigated. It is found that the source of this contribution is the interaction between the velocity components and the concentration deviations from its depth-mean value. The validation of the time-mean coefficient is made against the related numerical and experimental results. The effects of the second order dispersion coefficient and the dispersion coefficient oscillation on concentration transport are discussed.

1. Introduction

The effect of surface wave is an important factor needed to be considered in protection of ocean environments (Park et al., 2005). One of the influences of water waves is the longitudinal dispersion of mass concentration due to uneven distribution of Stokes drift and wave orbital velocity over water depth (Zeidler, 1976; Law, 2000). Different from tidal flows, a water wave has a significant vertical orbital velocity component apart from the horizontal one, both velocity components being comparable to each other, and having contributions to the dispersion coefficient. Such contributions result from the nonlinear interaction between wave-related flow velocities (Stokes drift and wave orbital velocity components) and the concentration deviation from depth-mean, as shown by the nonlinear nature of the advection term in the concentration transport equation. These contributions have not yet been studied in detail in the previous studies, which usually considered the effect of wave motions on the dispersion coefficient in an empirical way (Van Rijn, 1986), or on the base of laboratory experiments (Pearson et al., 2002), or by only including the effect of Stokes drift (Patil et al., 2007). From the view point of more accurate consideration of such contributions, a detailed investigation is needed. The present study makes such an effort in an analytical way.

The longitudinal dispersion of mass concentration due to uneven distribution of flow velocity over water depth has long been one of important subjects for riverine flows, and was also noticed to some extend for ocean flows for the case of the presence of surface waves. A resent research work for riverine flows can found in Chau (2000), which studied the variation of dimensionless transverse mixing coefficient with bottom roughness and shear velocity. For ocean flows, this problem is usually examined for a tidal current, which is treated as a long-period oscillatory shear flow (Smith, 1983; Yasuda, 1984). There are only limited research results relating to mixing process caused by surface waves. Dalrymple (1976) found that the mixing and transport of pollutants are enhanced by surface waves due to the drift velocity of a fluid particle and the presence of steady current in the direction of waves. There are also some observations which suggested that the magnitude of this longitudinal dispersion coefficient is small, except when the waves break. Holley et al. (1970) studied the dispersion coefficient for the flow containing steady and oscillating currents with linear vertical distribution and found that the contribution of oscillating current is negligible when oscillating period T is much smaller than the vertical diffusion time scale T_c ($T_c = h^2/D_z$ where h is the water depth and D_z is the vertical diffusivity). Law (2000) derived the dispersion coefficients caused by progressive surface waves by applying the N layers statistical model of Van Den (1990), and the longitudinal dispersion coefficient for a Stoke drift profile is established. He also examined the effect of the oscillatory orbital motion of pollutant particles induced by linear waves by a random walk numerical approach, but found that it is very small. This research only involved the passive motion of pollutants advected by flow velocity; the nonlinear interaction between wave-related flow velocity and the concentration deviation from depth-mean, illustrated by the nonlinear advection terms of the transport equation, was not considered. An example of this interaction was given by Yuan et al. (2004), who conducted the numerical solution to the vertical two-dimensional transport equation of concentration for velocity field of second order Stokes wave to investigate

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the effects of wave orbital velocity on the mean longitudinal dispersion coefficient, especially the effect of vertical orbital velocity. They showed that the contribution of the vertical orbital velocity component is up to 30% of the total mixing coefficient. The detailed interaction process was not shown in this research due to the limitation of numerical computation. For this purpose, a theoretical analysis is needed, and this is the motive of the present study.

The present research investigates theoretically the longitudinal dispersion coefficient induced by non-breaking waves described by the second order Stokes wave theory. The results are obtained by successive approximation. The first approximation presents the contribution of Stokes-drift, horizontal wave orbital velocity component and the interaction between them. The second approximation reveals the contribution of the vertical wave orbital velocity component and the contribution of the interactions between concentration deviation of the first approximation and horizontal wave orbital velocity component, in which the second order dispersion coefficient is also introduced. The third to fifth approximations give the contributions of the interactions between successive solutions obtained in the previous steps. Both mean and oscillating parts of dispersion coefficients are investigated. Apart from the water wave, the tidal current is also considered for a general result applicable to marine and coastal problems. The paper is organized as follows. Following Section 1, Section 2 introduces the definitions of the first and higher order dispersion coefficients. Section 3 describes the method of successive solution. Section 4 presents the results of successive approximations. Sections 5 and 6 give the result discussion and validation. The conclusions are drawn in Section 7.

2. Definition of dispersion coefficients

This section introduces the first and second order dispersion coefficients for the condition of surface waves, both of which include the mean and temporally and spatially oscillating parts. Consider the conservation of concentration on a vertical fluid volume column from bottom $z=-h$ to free surface $z=\eta$ in the (x,z) coordinate system with the z-axis pointing vertically upward and the x-axis laying at still water surface, as shown in Fig. 1, and then we have the depth-integrated one-dimensional transport equation for concentration c of the form

$$\frac{\partial}{\partial t}\int_{-h}^{\eta} c\,\mathrm{d}z+\frac{\partial}{\partial x}\int_{-h}^{\eta} uc\,\mathrm{d}z=\frac{\partial}{\partial x}\int_{-h}^{\eta} D_x\frac{\partial c}{\partial x}\,\mathrm{d}z \quad (1)$$

in which c is the concentration, t is the time, u is the horizontal fluid velocity component, and D_x is the horizontal turbulent diffusivity. We limit the present study to the case that velocity field is composed of tidal current, Stokes drift and first order wave orbital velocity. Then, the horizontal velocity component u in (1) can be written as $u=u_t+u_s+u_w$, where u_t is the tidal velocity, u_s is the Stokes drift velocity and u_w is the horizontal wave orbital velocity component.

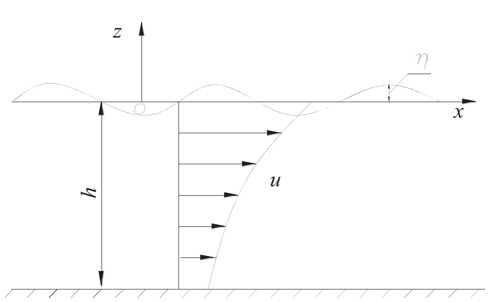

Fig. 1. Definition of the coordinate system.

To deal with the uneven distribution of velocity over water depth, we separate the velocity u and the concentration c into the depth-averaged parts (U,C) and the deviation parts $(\hat{u},\hat{c})$ from (U,C): $u=U+\hat{u}$, $c=C+\hat{c}$. Substituting into (1) yields

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}=\frac{1}{d}\frac{\partial}{\partial x}\left(dD_x\frac{\partial C}{\partial x}\right)-\frac{1}{d}\frac{\partial}{\partial x}\int_{-h}^{\eta}\hat{u}\hat{c}\,\mathrm{d}z \quad (2)$$

in which $d=h+\eta$ is the total water depth. To simplify (2), we introduce the first and higher order dispersion coefficients, D_1 and D_n $(n\geq 2)$, defined as

$$\int_{-h}^{\eta}\hat{u}\hat{c}\,\mathrm{d}z=d\sum_{n=1}^{\infty}(-1)^n D_n\frac{\partial^n C}{\partial x^n} \quad (3)$$

Among the higher order dispersion coefficient D_n $(n\geq 2)$, the present study only concerns the second order dispersion coefficient D_2. The physical meanings of the first and second order dispersion coefficients can be explained as follows. The first dispersion coefficient D_1 is the commonly used quantity describing the smoothing of concentration peak by mixing of water mass. The physical meaning of the second order dispersion coefficient can be seen in (3) and the resulting governing equations for C; it is multiplied by $\partial^2 C/\partial x^2$ in (3) and will appear in the governing equation for C as the coefficient of the term $\partial^3 C/\partial x^3$, as shown below.

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}=\frac{1}{d}\frac{\partial}{\partial x}\left(dK_1\frac{\partial C}{\partial x}\right)-\frac{1}{d}\frac{\partial}{\partial x}\left(dD_2\frac{\partial^2 C}{\partial x^2}\right) \quad (4)$$

where K_1 is the mixing coefficient incorporating the horizontal turbulent diffusivity D_x and the dispersion coefficient D_1: $K_1=D_x+D_1$. Eq. (4) is the governing equation for the depth-averaged concentration C, and from it we know that the difference of D_2 from D_1in physical meaning; it describes the smoothing of concentration peak by frequency dispersion; the concentration components with different temporally oscillating frequencies will propagate with different phase velocities. More discussions on this will be given in Section 5.3.

3. Solution method for concentration deviation $\hat{c}$

To determine the dispersion coefficients defined by (3), the concentration deviation $\hat{c}$ needs to be determined firstly. For this purpose, by substituting $u=U+\hat{u}$ and $c=C+\hat{c}$ into the two-dimensional advection-diffusion equation for concentration c ((1) is the depth integrated form of this equation) with the condition of $\partial C/\partial z=0$, the equation with unknown variables $\hat{c}$ and C can be obtained. Then, decomposing U into the mean part (wave-period-averaged parts) $\overline{U}$ and the oscillating part $\tilde{U}$, and transforming the equation into the moving coordinate system with $x'=x-\overline{U}t$, we can obtain the governing equation for concentration deviation $\hat{c}$ (the derivation process is omitted here for saving paper's space), which reads

$$\frac{\partial \hat{c}}{\partial t}-D_z\frac{\partial^2 \hat{c}}{\partial z^2}=-\left(\hat{u}\frac{\partial C}{\partial x}+\tilde{U}\frac{\partial \hat{c}}{\partial x}\right)+\left(\hat{u}\frac{\partial \hat{c}}{\partial x}-\left\langle \hat{u}\frac{\partial \hat{c}}{\partial x}\right\rangle\right)+\left(w\frac{\partial \hat{c}}{\partial z}-\left\langle w\frac{\partial \hat{c}}{\partial z}\right\rangle\right)+\frac{D_z}{h}\left[\frac{\partial^2 \hat{c}}{\partial z^2}\Big|_{z=0}\eta-\left(\frac{\partial \eta}{\partial x}\frac{\partial \hat{c}}{\partial x}\right)\Big|_{z=0}\right] \quad (5)$$

in which D_z is the constant vertical turbulent diffusivity, and the sign $\langle\ \rangle$ denotes depth-averaging as discussed below. Many field studies reported that there is a vertical diffusivity value, D_z, substantially larger than the molecular value (e.g. $D_z=10^{-4}$–$5\times 10^{-3}\ \mathrm{m^2/s}$ suggested in Wood et al. (1993)), indicating that turbulent eddy diffusion generally exists in the ocean. Although wave motion cannot generate vorticity and hence turbulence, the other physical processes, such as the current coexisting with the surface wave (the case considered here), wind-induced random breaking at the surface, thermal convection and flow separation,