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Surface gravity wave interaction with circular flexible structures

R. Mondal, S. Mandal, T. Sahoo *

Department of Ocean Engineering and Naval Architecture, Indian Institute of Technology, Kharagpur 721 302, India

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ABSTRACT

Surface gravity wave interaction with circular floating elastic plates of finite radius is analyzed in both the cases of single and two-layer fluids in finite water depth. The problems are analyzed under the assumption of small amplitude water wave theory and structural response. Further, gravity wave diffraction by circular elastic plates is studied under shallow water approximation based on linearized long wave theory. In the present analysis, the flexible structures are namely flexible circular plates and membranes of negligible draft. From the general formulation of the floating elastic plates, the results associated with the flexible membranes are obtained as special cases. Fourier–Bessel series type expansion formulae for the velocity potentials are obtained in the open water surface region and flexible plate/membrane covered region by the method of separation of variables. Suitable orthogonal mode-coupling relations are used along with the matching of velocity and pressure to obtain system of equations for the determination of the unknowns in the expansion formulae. Numerical results on structural deflection are computed and plotted to understand the hydrodynamic characteristics of the floating structures under water action. In order to understand the flow distribution around the floating circular flexible structures, contour plots are provided.

1. Introduction

There has been a significant progress on the study of surface wave interaction with very large floating structures of various configurations and geometry. The success of the mega-float project in Japan has created technological confidence to build such large floating structure. For optimum utilization of ocean space and for future expansion activities, often circular structures are considered. Further, in the marginal ice zone (MIZ), ice floes are observed which are formed due to the freezing or by the breaking of the outer perimeter of the inner land first ice. These floating ice sheets are of different shapes and geometry. For simplicity, often these ice sheets are considered as circular flexible disk like structure of uniform thickness. In many situation, circular floating flexible structures are used as breakwater in costal region to protect varies marine facilities.

The study on wave interaction with very large floating structures and floating ice sheets is based on the linearized theory of water waves and small amplitude structural responses in which the floating structure and ice sheet are modeled as Euler–Bernouli beam equation (Wang and Tay, 2011). Motivated by the proposal to build an artificial island offshore from the Scripps Institution of Oceanography, Garrett (1971) studied the wave scattering by a rigid circular dock of finite draft using the Fourier–Bessel series type expansion of the velocity potentials. Meylan and Squire (1996) used two mathematical methods to study the response of a circular ice floe in three dimensions in which the ice-floe is modeled as a thin flexible plate. In case of finite water depth, Peter et al. (2004) derived a closed form solution for the wave scattering by a circular floating plate by matching the velocity and pressure at the interface of the circular plate and the water and using the orthogonal characteristic of the vertical eigenfunctions in the open water surface region. Andrianov and Hermans (2005) used the Green's function technique to study the hydroelastic analysis of a circular plate floating on the free surface in both the cases of finite and infinite water depths. Squire (2008) reviewed the synergies between the hydroelastic analysis of very large floating structures and floating ice sheet. Bennetts et al. (2009) studied the wave scattering by circular ice floe in case of finite water depth. Bennetts and Squire (2009) studied the wave interaction with circular floes of varied characteristics in multiple rows. Further, Bennetts and Williams (2010) analyzed the wave scattering by ice floe of arbitrary shape. Recently, Montiel et al. (2013) analyzed the hydroelastic response of floating circular disc and compared their theoretical results with the data obtained from the wave basin experiments.

Apart from the study on small amplitude water wave theory, there has been an extensive study on wave structure interaction problems under shallow water approximation. Sturova (2001)

* Corresponding author.
E-mail addresses: tsahoo1967@gmail.com, tsahoo@naval.iitkgp.ernet.in (T. Sahoo).

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studied the diffraction of surface waves by a plate of arbitrary geometry floating on the free water surface using boundary integral equation method. Meylan (2002) derived the spectral theory to study the time dependent wave motion of a thin floating plate in shallow water. Andrianov and Hermans (2003) performed the hydroelastic analysis of the very large floating structures under small amplitude wave theory as well as shallow water approximation. Sturova (2006) analyzed the vibration of a rectangular floating flexible plate under the action of periodic localized force. Sturova (2009) investigated the unsteady behavior of elastic articulated floating flexible plates. The above discussions are based on the Cartesian coordinate system. Motivated by the suggestion for designing a circular floating island as one of the alternate geometries along Israeli coast, a closed form solution was developed by Zilman and Miloh (2000) to study the wave diffraction by a flexible circular plate without compressive force under shallow water approximation. Sturova (2003) studied the transient problem of wave interaction with floating circular plate in the presence of external load. As mentioned above, the problems were discussed in homogenous fluid medium.

In real ocean, often water density changes due to the variation in water depth. This occurs due to the variation of temperature and salinity. Recently, there is a significant interest on wave structure interaction problems in two-layer fluid having a common interface. Linton and McIver (1995) studied the interaction of surface waves in two-layer fluid such that upper layer of finite depth and lower-layer of infinite depth and a cylinder is horizontally submerged on either of the fluid layers. Cadby and Linton (2000) generalized the study of Linton and McIver (1995) from two dimensions to three dimensions. Bhattacharjee and Sahoo (2008) derived the expansion formulae for the velocity potential associated with the flexural gravity waves in a two-layer fluid in a two-dimensional fluid domain having a plate covered surface and an interface in both the cases of water of finite and infinite depths. Mondal and Sahoo (2012) generalized the expansion formulae for the velocity potentials derived by Bhattacharjee and Sahoo (2008) to deal with flexural gravity wave problems in three dimensional fluid domain and obtained various characteristics of the associated eigenfunctions and demonstrated the utility of the expansion formulae by analyzing flexural gravity wave scattering due to a crack in a floating ice sheet. These problems in two-layer fluid are analyzed in the Cartesian coordinate system. However, to the authors' knowledge, no study is available in the literature on wave interaction with circular floating elastic plate in two-layer fluid.

Schulkes et al. (1987) studied the effect of compression on flexural gravity wave motion. Balmforth and Craster (1999) studied wave interaction with floating ice sheet in the presence of compressive force taking the ice thickness into account. Mondal et al. (2013) studied the effect of compressive force on the wave scattering by a crack in a floating ice sheet in homogenous fluid medium. Mohapatra et al. (2013) studied the effect of compression on diffraction of waves by a finite floating elastic plate using boundary integral equation method.

Apart from wave interaction with flexible floating structures as discussed above, there is a parallel interest on wave interaction with floating/submerged membrane. The flexible membranes are important in the marine industry for their use as vertical or floating breakwaters to protect marine facilities from incident wave load. The advantages of these flexible membranes are that these structures are easy to carry, inexpensive, reusable, rapidly deployable and removable. The performance of a vertical-screen membrane breakwater was investigated by Williams (1996) and the literature cited therein. The performance of a vertical-screen membrane breakwater was investigated by Kee and Kim (1997). Cho and Kim (2000) studied the monochromatic incident wave interaction with a horizontal porous flexible membrane using two dimensional linear wave theory. Lee and Lo (2002) derived the analytical solution for wave interaction with flexible membrane under the assumption of linear water wave theory. Karmakar and Sahoo (2008) derived the expansion formulae for wave membrane interaction for the step bottom topography in both the cases of finite and infinite water depths. Recently, Karmakar and Soares (2012) studied the wave scattering by floating moored membrane in case of shallow water considering the linear water wave theory. Mandal et al. (2013) studied the hydroelastic response of a three-dimensional porous flexible concentric cylinder system in regular waves in finite water depth based on small amplitude water wave theory using Fourier–Bessel Series expansion method and least square approximation.

In the present work, surface gravity wave interaction with floating flexible circular structures are analyzed in both the cases of single-layer and two-layer fluids in finite water depth under small amplitude water wave theory and shallow water approximation. The flexible structures considered in the present study are the floating (i) flexible plate under the action of uniform compressive force and (ii) flexible membrane under the action of tensile force. The problems are analyzed using Fourier–Bessel series type of expansion formulae along with certain generalized orthogonal mode-coupling relations as appropriate. The solution procedures for wave interaction with flexible structures are demonstrated in case of flexible plates and membranes. In addition, the long wave equations inn single and two-layer fluids in the presence of the floating flexible circular plate and membrane are derived under shallow water approximations. Numerical results are demonstrated in different cases to understand the effects of various physical parameters on the deflection of the structures.

2. Wave interaction with floating flexible circular structures

In the present section, surface gravity wave interaction with floating circular flexible plate and membrane are studied in both the cases of single and two-layer fluids under the assumption of small amplitude water wave theory in water of finite depth. The floating flexible plate is modeled based on the assumption of thin plate theory which is acting under uniform compressive force having free edge and the membrane is modeled as a thin inextensible sheet of mass density acting under uniform tension. The mathematical problem is modeled as a boundary value problem in terms of the velocity potentials in the cylindrical polar coordinate system. The velocity potentials are obtained in terms of Fourier–Bessel series for both the open water region and floating structure covered region. Using generalized orthogonal mode-coupling relations in the plate and membrane covered region and the matching conditions at the interface of the fluid structure boundary, the unknown constants in the expansion formulae are obtained. The problems in case of single and two-layer fluids are handled for solution followed by certain numerical results in the subsequent subsections.

2.1. Wave interaction with flexible structures in single-layer fluid

The problem is considered in the cylindrical polar coordinate system (r, θ, y) with r–θ plane being the horizontal plane and the y-axis being in the vertically downward direction. It is assumed that a thin circular plate of uniform thickness d and radius a is floating on the mean free surface of a fluid medium of uniform density ρ and of finite depth h as in Fig. 1.

The draft of the circular plate is considered as zero for simplicity with origin of the coordinate system being the center of the circular plate on the undisturbed mean free surface. In the presence of floating circular plate, the fluid domain is divided into