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A two-time scale control law based on singular perturbations used in rudder roll stabilization of ships

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ABSTRACT

A two-time scale decomposition method is used to analyze and design the rudder roll stabilization (RRS) system of ships. In the surge-sway-roll-yaw ship motion system, roll motion is much faster than the others, the interactions between these fast and slow dynamics cause the non-minimum phase behavior in roll dynamics, which is regarded as a major challenge in RRS control design. A small parameter ε is introduced to describe the fast roll dynamics by a singular ordinary differential equation. By using singular perturbation approaches, the system is then decomposed into two different time scale subsystems, a quasi-steady-state subsystem to describe the slow dynamics, and a boundary layer subsystem to describe the fast dynamics. Separate control strategy is used to stabilize each subsystem and the coupling effect between the subsystems is also considered. A Lyapunov function is constructed for the slow subsystem and robust analysis is made to evaluate the unmodeled dynamics. Simulation results show the effectiveness and robustness of this approach used in RRS system.

1. Introduction

Due to the relatively small moment of inertia compared to other degrees of freedom (DOFs), the roll motion of a surface ship is easily affected by the environmental disturbances such as waves and wind, and often produces the largest acceleration. Large roll motion is the main cause of seasickness, can greatly affect the comfort of the passengers, decrease the work efficiency of the crew, damage the cargo, and in some extreme cases, may cause the capsizing of the ship. Therefore, ship roll reduction has become an active research area since 1970s. Criteria of the maximum roll angle for different work conditions have been made by Faltinsen (1993). It is suggested that the maximum root mean square of roll angle should be less than six degrees for light manual work and three degrees for intellectual work.

In the past decades, many devices have been designed to reduce the roll motion, both active control and passive control devices, such as bilge keels, gyroscopic stabilizers, anti-rolling tanks, stabilizing fins and moving weights (Treakle et al., 2000; Gawad et al., 2001; Perez and Blanke, 2002; Townsend et al., 2007; Surendran et al., 2007). However, all these approaches need extra devices and installation costs, thus are usually expensive.

Although the original objective of the rudder is to steer the ship to a desired course, for most surface ships, rudder action can also cause certain roll motion. So it is expected that if the rudder is suitably operated according to the roll motion and the course deviation, the roll angle may be reduced to some degree, at the same time the heading is not violently changed. This rudder roll stabilization (RRS) control strategy needs no extra devices and is relatively cheap, thus has drawn many researchers' interests in the past decades (Van Amerongen et al., 1990; Blanke and Christensen, 1993; Lauvdal and Fossen, 1998; Perez, 2005). Model experiments and full-scale trails have been made to evaluate its effectiveness in practice (Van Amerongen et al., 1990). In RRS control system, rudder is the only actuator for two outputs (roll and heading), thus sufficient bandwidth separation of the two loops has to be guaranteed.

There are also several drawbacks of RRS, such as the inefficiency at low speed and severe feedback limitations due to rudder saturation and rate limits. Besides, it is well-known that ships have non-minimum phase (NMP) behavior in the rudder-to-roll dynamics, which is considered to be one major challenge for RRS (Lauvdal and Fossen, 1997; Perez, 2005). NMP systems have an inverse initial response and large phase lag. The NMP behavior in roll motion often causes a fundamental limitation in the RRS system: disturbances attenuation at some frequencies will result in amplification at other frequencies. This limitation thus poses a

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trade-off between reducing the roll angle at certain frequencies and amplification at others (Perez, 2005).

The NMP phenomenon often arises from the interaction between opposite fast and slow dynamic effects in the system (Perez, 2005). As to the ship, the NMP behavior in roll motion is caused by the fact that the roll dynamics is much faster than the other DOFs. Singular perturbation approach is such a method to analyze and separate the different time scale motions in control problems. In this paper, the RRS system for ships is decomposed into two different time scale subsystems, namely the quasi-steady-state (slow) subsystem and boundary layer (fast) subsystem. The control objectives and control strategies of the two subsystems are treated separately.

Singular perturbation approaches have been used in aerospace industry for many years as a time-scale separation technique (Mehra, 1979; Bertrand et al., 2011; Esteban et al., 2013). For example, a three-time scale control law is designed for a nonlinear helicopter model in vertical flight (Esteban et al., 2013). This can be done due to three different time scales of altitude motion, angular velocity, and the associated collective pitch angle of blades. A comprehensive literature review of singular perturbation used in aircraft control was made by Naidu and Calise (2001). However, despite of the extensive work in aerospace industry, few work of singular perturbation and time scale separation techniques has been done in ship control community. This is mainly due to the relatively poor rudder effect and simple control objectives for a ship control system. However, when a RRS problem is considered, the traditional 3-DOF model (surge-sway-yaw) is coupled with fast roll motion, and different time scale motions do exist in this system. The concept of time scale separation based on singular perturbation can be used to analyze such problems in a natural and elegant way.

Singular perturbation is a means of taking into account the often neglected high-frequency phenomena and considering them in a separate fast time scale (Kokotovic et al., 1987). By introducing a small parameter ε, the fast varying state variables are described in the form of singular ordinary differential equations (ODEs), the equations become singular when ε tends to zero. A stretched time scale is used to describe the fast dynamics and the slow state variables are regarded to be constant in this time scale. A so-called quasi-steady-state equilibrium (QSSE) is used to pass information between different time scale subsystems.

This paper introduces the singular perturbation approach to analyze the ship RRS problem. There are three major merits of using this approach in RRS system.

Firstly, more detailed analysis is possible in time domain, such as stability issues and time domain response. Unlike traditional analysis methods, whose emphasis is on the bandwidth separation in Bode diagram considered in frequency domain, this paper emphasizes the separation of different time scale subsystems in time domain. The stability and robust analysis are easily conducted in this model, and the sensitivity analysis to model errors can also be evaluated within this framework, which are not easily conducted in frequency domain.

Secondly, the time scale decomposition approach and separate control strategy simplify the control law design for RRS system. By using singular perturbation method, the original underactuated RRS system can be decomposed into two single input single output (SISO) subsystems, thus is relatively easier to obtain the appropriate control law that can stabilize each subsystem. Thirdly, the proposed separate control strategy considers the interaction between different time-scale subsystems. The coupling effect is important in some cases, and singular perturbation approach takes this into consideration though QSSE.

In this paper, a simplified 3-DOF (sway-roll-yaw) linear model is used to design and analyze the RRS control law. As course keeping operations are considered in most situations, the linear model has considerable accuracy in these problems (Perez, 2005). Li et al. (2009) used a comprehensive 4-DOF (surge-sway-roll-yaw) nonlinear model as a virtual ship for simulation and performance evaluation. This nonlinear model was obtained by a set of captive model tests (Son and Nomoto, 1982). It is selected as a benchmark model to evaluate the performance of the linear model in this paper. The different performances between the linear and nonlinear models are evaluated.

The structure of this paper is as follows. Section 2 introduces the nonlinear and linearized models of motion of surface ships, the model of disturbances is also described. Section 3 gives a brief introduction to the singular perturbation approach, based on which the RRS control is designed. Robustness analysis of the unmodeled dynamics is also made in this section. Section 4 gives the simulation results. Section 5 is the conclusion.

2. Model definition and analysis

In this section, the models of ship motion and environmental disturbances are described.

2.1. 4-DOF nonlinear model

A ship in a seaway moves in 6-DOFs. Three translation displacements are used to define the location and three angular displacements are used to define the orientation. These motions are often described in two types of reference frame, namely the inertial frame and body-fixed frame.

As shown in Fig. 1, the location and orientation of the ship are described in the inertial frame, the translation displacements and angular displacements are described as $[x_0, y_0, z_0]^T$ and $[\phi, \theta, \psi]^T$, where x_0, y_0 and z_0 are the three coordinates of the ship, ϕ, θ and ψ are roll, pitch and yaw angle, respectively. The components of the force and moment $[X, Y, Z]^T$, $[K, M, N]^T$, the components of the translational velocity and the angular velocity $[u, v, w]^T$, $[p, q, r]^T$, are described in the body-fixed frame, where u, v and w are surge, sway and heave velocity, and p, q and r are roll, pitch and yaw rate, respectively. The rudder angle is expressed as δ.

In traditional maneuvering issues, such as course-keeping problem, normally only a 3-DOF model (surge-sway-yaw) is considered. However, when consider the RRS problem, a 4-DOF model including the roll motion is needed. In this paper, a comprehensive 4-DOF nonlinear model (surge-sway-roll-yaw) is used to describe the RRS system (Fossen, 1994):

$$(m+m_x)\dot{u}-(m+m_y)vr=X \tag{1}$$

$$(m+m_y)\dot{v}+(m+m_x)ur+m_y\alpha_y\dot{r}-m_yl_y\dot{p}=Y \tag{2}$$

$$(I_x+J_x)\dot{p}-m_yl_y\dot{v}-m_xl_xur+W\overline{GM}\phi=K \tag{3}$$

$$(I_z+J_z)\dot{r}+m_y\alpha_y\dot{v}=N-Yx_G \tag{4}$$

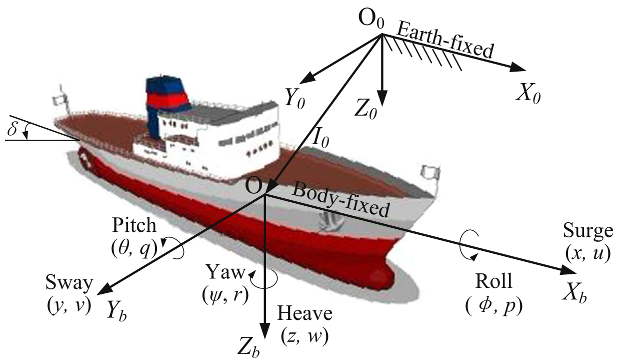

Fig. 1. Ship motion in 6-DOF.