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Analytical solutions for oscillations in a harbor with a hyperbolic-cosine squared bottom

Gang Wang [a,b,*], Jinhai Zheng [a,b,*], Qiuhua Liang [b,c], Yanna Zheng [d]

[a] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing 210098, China
[b] College of Harbor Coastal and Offshore Engineering, Hohai University, Nanjing 210098, China
[c] School of Civil Engineering & Geosciences, Newcastle University, Newcastle upon Tyne NE1 7RU, UK
[d] School of Ocean and Civil Engineering, Dalian Ocean University, Dalian 116034, China

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ABSTRACT

Based on the linear shallow water approximation, longitudinal oscillations in a rectangular harbor with a hyperbolic-cosine squared bottom induced by incident perpendicular waves are analytically investigated, which could be described by combining the associated Legendre functions of the first and second kinds. The effects of topographic parameters on the resonant spectrum and response are examined in detail. When the width of the harbor is of the same order magnitude as wavelengths, transverse oscillations may exist due to the wave refraction. Analytic solutions for transverse oscillations within a harbor of hyperbolic-cosine squared bottom are derived. These oscillations are typically standing edge waves. The transverse eigenfrequency is found to be related not only to the width of the harbor, but also to the varying water depth parameters.

1. Introduction

Better understanding of wave trapping and oscillations in bays and harbors is of importance to many practical applications. These oscillations may be induced by tsunamis, infragravity waves, atmospheric fluctuations and variable currents traveling into a semi-enclosed domain, e.g. bays and harbors (Bellotti et al., 2012; De Jong and Battjes, 2004; Fabrikant, 1995; Okihiro and Guza, 1996). The oscillations may cause unacceptable vessel movements, affect normal operation of docks, and generate excessive mooring forces that may break the mooring lines. Rabinovich (2010) reviewed recent advances in understanding and modeling of seiches and harbor oscillations. In order to reduce the disturbance to normal harbor operation and minimize the possible damages, a further research effort is necessary to enhance our current knowledge for this type of wave amplification and its excitation mechanisms and thus improve predictive capability.

Numerical modeling has provided an effective way to reproduce such a phenomenon and identify the eigenvalues of oscillations, through simulating waves propagating from offshore and subsequently being amplified inside a bay/harbor. Researchers have developed a variety of numerical models based on the mild-slope equations to predict wave oscillations in harbors induced by different offshore wave conditions (Bellotti, 2007; Maa et al., 2011; Panchang et al., 2000). These linear models are useful for predicting short wave disturbances in harbors and identifying harbor resonance periods and long wave amplification factors. Furthermore, this type of models is generally computationally efficient and thus suitable for applications to large computational domains; this facilitates explicit account of possible effects of ambient wave transformation outside the harbor in a larger region. However, these models cannot reliably predict long-period oscillations induced by incident short waves or higher harmonics generated from nonlinear interactions. Boussinesq-type models provide a more reliable tool for simulating such nonlinear hydrodynamic problems including the nonlinear generation of long waves by groups of short waves propagating from deep to shallow water, diffraction of both short- and long-period waves into a harbor, and resonant amplification of long waves inside a harbor. They represent a group of models based on extended and higher-order Boussinesq equations that are solved using the finite difference method on structured Cartesian (Shi et al., 2012; Zou and Fang, 2008) or curvilinear meshes (Shi et al., 2001) or the finite element method on triangular or quadrilateral grids (Losada et al., 2008; Walkley and Berzins, 2002). Most of these numerical models are capable of capturing the major characteristics of the resonant response from a given geometry and hence are useful for designing plan shape of harbors. However, designing a new geometry generally involves a large number of variables, including the size of the basin and the width of the entrance, among others.

* Corresponding authors at: State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing 210098, China.
Tel.: +86 83787706.
E-mail addresses: gangwang@hhu.edu.cn, wanggang1015@mail.dlut.edu.cn (G. Wang), jhzheng@hhu.edu.cn (J. Zheng).

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The entire design process may therefore require numerous computationally demanding model runs, which may not be feasible in practice. Theoretical analysis provides an alternative means for investigating the resonant mechanism being influenced particularly by the geometry parameters, topography effects, generation forcing, etc.

Miles and Munk (1961) first considered the enhancement in harbor surging in a rectangular harbor with narrow openings, matching the normal velocity and water surface elevation at the entrance with a scattering theory. Lee (1971) introduced the 'arbitrary-shape harbor' theory by applying the Weber solution of the Helmholtz equation in the regions both inside and outside the harbor, with the final analytical solution obtained by matching the wave amplitudes and their normal derivatives at the entrance. Mei and Ünlüata (1976) derived the analytical solutions of resonance for a harbor with two coupled rectangular basins subjected to periodic incident waves. Based on the multiple-scales perturbation method, Wu and Liu (1990) obtained the analytical solutions for the second-order low-frequency oscillations inside a rectangular harbor excited by incident wave groups. Yu (1996) examined parametrically the dissipative effects of a river channel on bay oscillations under resonant states. Fabrikant (1995) reported analytical solutions for harbor oscillations as a result of linear instability in a system featured by coupled surface-wave resonator and shear flow. Although these analytical studies have been restricted to the wave-induced oscillations in simple-geometry harbors with horizontal bottom, the general features of resonance have been considered and represented.

Harbor resonance is also strongly dependent on the topography. But most of the previous theoretical analysis only pays attention to the change in height as the wave shoals (Mattioli, 1978; Zelt and Raichlen, 1990). Wang et al. (2011a) presented new formulations for describing longitudinal oscillations inside a harbor with constant slope and further demonstrated that different transverse oscillation modes may exist due to the wave refraction. They further proved that these transverse oscillations can be induced by normal-incident waves due to instability and seafloor movements in the harbor (Wang et al., 2011b, 2013).

The beach has been represented not only by a straight slope, but also by exponential profiles (Buchwald and Adams, 1968; Clarke, 1973). The present study will examine oscillations within a harbor of a convex hyperbolic-cosine squared bottom, where the profile only varies in offshore direction and it can be described as $h=h_0\cosh^2(\lambda x)$. By appropriately setting values of the constants, this model can also represent actual beaches, as shown in Fig. 1.

There are assumed initially only longitudinal oscillations within a rectangular harbor of a hyperbolic-cosine squared bottom and their formulary descriptions are given in Section 2. When the width of a harbor is of the same order magnitude as the wavelength, transverse oscillations would be present, and their analytic formulations are given in Section 3. Conclusions are drawn in Section 4.

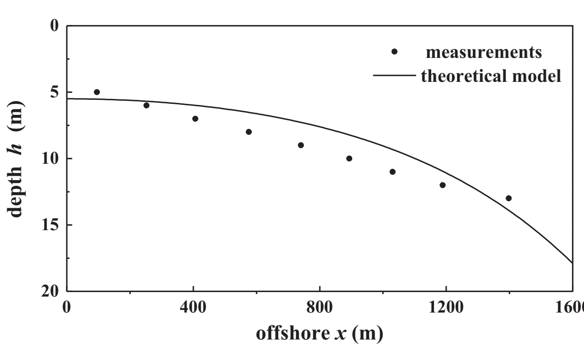

Fig. 1. Comparison of the bottom topography of the east beach near Marina di Carrara, and the constants for the theoretical model are $h_0=-5.5$ m and $\lambda=0.0008\ \mathrm{m}^{-1}$. The topographic data are extracted from the paper of Bellotti and Franco (2011).

2. Longitudinal oscillations

2.1. Formulation and solution method

Fig. 2 illustrates an idealized rectangular harbor with a hyperbolic-cosine squared bottom, located from $x=0$ to $x=L$. The x axis is normal to the backwall and the y axis is parallel to the backwall with the z axis vertically downwards from the still water level. The width of the harbor is $2b$ (from $y=b$ to $y=b$), which is assumed to be much smaller than the wavelength of the incident waves so that only longitudinal oscillations occur. Assuming horizontal seafloor in the open ocean, the water depth is thus given by

$$h(x,y)=\begin{cases} h_0\cosh^2(\lambda x) & 0\le x\le L,\ \ |y|\le b \\ h_1 & x>L \end{cases} \tag{1}$$

where λ (m^{-1}) is a parameter determining the shape of the hyperbolic-cosine squared, and

$$h_1=h_0\cosh^2(\lambda L) \tag{2}$$

In most harbors, water is relatively shallow compared with the oscillation wavelength. So the motion of water particles is predominantly horizontal and the vertical variation is weak, which satisfies the assumptions of the shallow water equations.

According to the linear shallow-water approximation, the surface elevation $\eta(x, y, t)$ satisfies the following equation:

$$\eta_{tt}-g\nabla(h\nabla\eta)=0 \tag{3}$$

in which $\nabla=(\partial/\partial x, \partial/\partial y)$, g is the acceleration due to gravity and t is the time.

The water surface displacement in the harbor may be written as

$$\eta_I^L(x,t)=\zeta_I^L(x)\exp(i\omega t) \tag{4}$$

where the subscript I indicates 'inside the harbor', the superscript L represents longitudinal effects, and $i=(-1)^{1/2}$. Eqs. (1), (3) and (4) are combined to yield

$$\frac{d^2\zeta_I^L}{dx^2}+2\lambda\tanh(\lambda x)\frac{d\zeta_I^L}{dx}+\frac{\omega^2}{gh_0}\mathrm{sech}^2(\lambda x)\zeta_I^L=0 \tag{5}$$

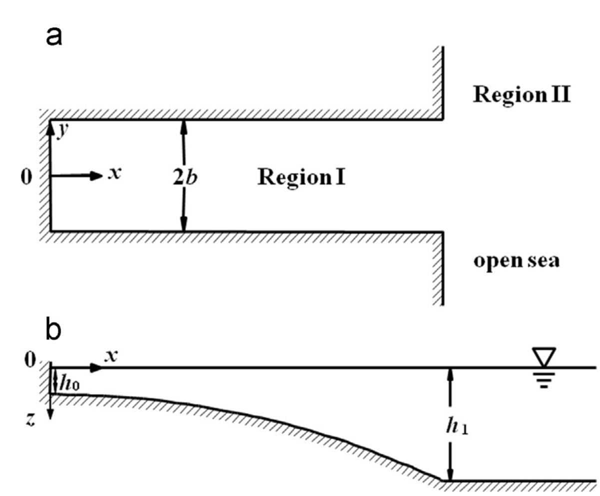

Fig. 2. Definition sketch of the rectangular harbor with a hyperbolic-cosine squared bottom. (a) plan view and (b) elevation view.