Ocean Engineering 85 (2014) 12–20

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

A fast approach coupling Boundary Element Method and plane wave approximation for wave interaction analysis in sparse arrays of wave energy converters

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ARTICLE INFO

Article history:
Received 25 July 2013
Accepted 19 April 2014
Available online 9 May 2014

Keywords:
Wave interactions
Wave energy converter
Boundary Element Method
Plane wave approximation
Power production

ABSTRACT

A computational approach is developed to investigate wave interaction effects in sparse arrays of floating bodies (such as wave energy converters) based on linear potential theory. In particular, the wave diffraction and radiation problems in a multiple body array are solved in reasonable time and accuracy. In contrast to previous approaches that have considered all bodies in the array as a single module, the present approach treats each body in the array as an isolated body. The interactions resulting from the scattered wave field among the bodies are then taken into account via plane wave approximation in an iterative manner. The boundary value problem corresponding to an isolated body is solved by the Boundary Element Method (BEM). The approach is useful for wave periods in the range 4–15 s, provided that the bodies are separated by at least five times the characteristic dimension of a body. The main advantage of the approach is that the computational time and memory requirements are significantly less than that of conventional BEM. In this paper, first, the numerical results for hydrodynamic coefficients computed by the proposed approach are validated against conventional BEM. Next, the wave interaction effects on power production are investigated in arrays of 50 wave energy converters.

1. Introduction

This study investigates wave interaction effects in arrays of oscillating wave energy converters (WECs). It is anticipated that for power production at a commercial scale, tens of WECs would be deployed in arrays, the so-called WEC farms. Therefore, in order to make reliable estimates of energy production, it becomes necessary to take into account wave interaction effects. This aspect has been covered in many studies, c.f. Budal (1977), Falcaode (2002), Cruz et al. (2009), Ricci et al. (2007), Babarit (2010) and Borgarino et al. (2012), in regular and irregular wave fields for a variety of WECs. In developing oscillating WEC arrays with device dimensions ~10–20 m, the distance between the devices could be as much as a few hundred meters due to practical considerations, such as mooring, installation and maintenance issues. Consequently, this study investigates wave interaction effects in such sparse WEC arrays, where the ratio of separating distance to device dimension is large.

A viable approach to solving full hydrodynamic interaction phenomena in a multiple body array is to model the problem using linear potential theory. This requires solving the diffraction and radiation problems over the frequency domain; the hydrodynamic coefficients (excitation force, added mass and wave damping) can then be easily post-processed. A judicious choice for solving these problems is the Boundary Element Method (BEM), since the domain is unbounded and only discretization of the boundary is required. However, when the array consists of several bodies, the number of discretization elements increases, and use of the BEM becomes prohibitive due to the computational requirements of solving a dense linear matrix system. In addition, a comprehensive analysis requires that the solution of diffraction and radiation problems be sought at several frequencies in the range of interest, thereby further increasing the computational time requirements. It is worth noting that developing fast algorithms to solve dense matrix system in BEM is an active area of research. There have been successful attempts to accelerate the BEM by coupling it with fast methods, such as fast multipole methods (FMM) (Greengard and Rokhlin, 1987), pre-corrected fast Fourier transforms (FFT) (Phillips and White, 1997) and other methods. In the field of wave hydrodynamics, the BEM–FMM coupling for solving diffraction and radiation problems has been used in Utsunomiya and Watanabe (2002), Teng and Gou (2006), and Borgarino et al. (2011) for specific applications. Acceleration using fast Fourier transforms has been performed in Kring et al. (2000) and others. However, these approaches have some limitations. In FMM,

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http://dx.doi.org/10.1016/j.oceaneng.2014.04.019
0029-8018/

numerical convergence of the existing multipole series expansion of the free surface Green's function can be difficult, thereby limiting the scope of developing this approach for generic three-dimensional bodies; whereas, in FFT methods, construction of a grid and projection operations over the full domain of the sparse array would lead to a sub-optimal algorithm.

Besides the aforementioned fast methods, interaction approaches have also received a significant attention, particularly for axisymmetric bodies. A multiple scattering theory (Ohkusu, 1974) combined with a direct matrix method has been developed in Kashiwagi (2000). An extension of this theory, the so-called hierarchical interaction theory, has been presented in finite (Kagemoto and Yue, 1986) and infinite depths (Peter and Meylan, 2004). The plane wave approximation (PWA), also known as wide space approximation, coupled with direct matrix approach has been proposed in Simon (1982). The PWA approach is based on the assumption that waves diffracted by or radiated from a body can be approximated at large distances by a plane wave. This approach has been further improved to reduce the spacing requirements by taking into account non-plane correction terms in the formulation (McIver and Evans, 1984). A comparison of the multiple scattering and the PWA approaches for arrays consisting of five vertical cylinders with varying radii and separating distances has been presented in Mavrakos and McIver (1997), in which it was deduced that the PWA provides very good results for all the hydrodynamic coefficients in comparison to those provided by multiple scattering approach when the ratio of separating distance to body dimension is greater than five.

In this study, we propose a simple approach for estimating the wave interaction effects in sparse arrays of a large number of bodies of arbitrary shape. The approach couples the efficiency of the BEM with the core idea of PWA mentioned above. Essentially, when the bodies are sufficiently distant in an array, the effects of the wave field emanating from one body are taken into account by the other body as an additional plane incident wave. Here we use this approximation efficiently and develop a fast approach. The computational time and memory requirements using the proposed approach are significantly less than the conventional BEM (CBEM), and the approach is applicable to bodies of arbitrary shape. Furthermore, the approach is simple and can be easily implemented in existing diffraction/radiation solvers.

The remainder of the paper is organized as follows. We begin by recalling in Section 2 the problem formulation in potential theory and detail the necessary equations that must be solved to calculate power extraction by a WEC array. In Section 3, we illustrate our approach of coupling BEM with PWA, and in Section 4 we present numerical results to validate the approach. Section 5 analyze wave interaction effects on power production in WEC arrays. The final section presents some concluding remarks.

2. Problem statement

The wave interaction phenomenon in a multiple body array is modeled within the framework of linearized potential theory. Specifically, the fluid is inviscid and incompressible and the flow is irrotational. The wave amplitude and body motions are small with respect to the wavelength and body dimensions, respectively. Under these assumptions, the problem can be formulated in terms of a velocity potential, Φ, satisfying the Laplace equation in the fluid domain with appropriate boundary conditions. For simplicity, we consider that the fluid domain is of infinite depth and unbounded in horizontal directions. The motion is time harmonic with circular frequency ω, i.e. $\Phi(x,y,z,t)=\Re\{\phi(x,y,z)e^{-\mathrm{i}\omega t}\}$. Linearization allows the velocity potential ϕ to be expressed as sum of the incident potential ϕ_{in}, the diffraction potential ϕ_d and the radiation potential ϕ_r. The explicit form of the incident potential is

$$\phi_{in}=\frac{gA}{\omega}e^{kz}e^{\mathrm{i}k(x\cos\beta+y\sin\beta)} \tag{1}$$

where $k=\omega^2/g$ is the wave number, g is the acceleration due to gravity, A is the wave amplitude and β is the angle between the direction of propagation of the incident wave and the positive x-axis. The diffraction and radiation potentials correspond to the potentials generated in response to the incident waves and the fluid disturbance due to the motions of the bodies in still water, respectively. The diffraction and radiation boundary value problems can be summarized as follows:

- Diffraction problem:

$$\begin{cases} \Delta\phi_d=0 & \text{in the fluid domain} \\ \nabla\phi_d\to 0 & z\to-\infty \\ \dfrac{\partial\phi_d}{\partial z}-k\phi_d=0 & \text{at mean free surface position } z=0 \\ \dfrac{\partial\phi_d}{\partial n}=-\dfrac{\partial\phi_{in}}{\partial n} & \text{on mean wetted body surface} \end{cases} \tag{2}$$

- Radiation problems:

$$\begin{cases} \Delta\phi_{r_i}^j=0 & \text{in the fluid domain} \\ \nabla\phi_{r_i}^j\to 0 & z\to-\infty \\ \dfrac{\partial\phi_{r_i}^j}{\partial z}-k\phi_{r_i}^j=0 & \text{at mean free surface position } z=0 \\ \dfrac{\partial\phi_{r_i}^j}{\partial n}=n_i^j & \text{on mean wetted body surface} \end{cases} \tag{3}$$

where indices i and j correspond to motion in any of the six degrees of freedom and the numeration of the body ($j=1,\dots,N$), respectively. Using this convention, n_i^j denotes the component of the normal vector in the direction of motion on body j.

Having solved the set of diffraction and radiation problems, it is straightforward to compute the hydrodynamic coefficients: excitation force $F_{ex}(\omega)$, added mass $AM(\omega)$ and wave damping $B(\omega)$. Further, to compute the motion of a system of N floating bodies for unit wave amplitude and wave frequency ω we solve

$$(M+AM(\omega))\ddot{X}+(B_{PTO}+B(\omega))\dot{X}+(K_H+K_{PTO})X=F_{ex}(\omega) \tag{4}$$

where X is the position vector, $X=\Re(\overline{X}e^{-\mathrm{i}\omega t})$, $\dot{X}$ and $\ddot{X}$ being, respectively, the velocity and acceleration vectors. M and K_H are the mass and hydrostatic matrices of the system. An idealized power take off (PTO) is considered in this study, composed of a linear spring and damper system with stiffness k_{PTO} and damping coefficient b_{PTO}. These are the diagonal elements of the stiffness and damping matrices in (4) , i.e. $K_{PTO_{ii}}=k_{PTO}$ and $B_{PTO_{ii}}=b_{PTO}$.

In regular waves, the mean power extracted by each device in the array per unit square of wave amplitude is

$$p_i(\omega)=\tfrac{1}{2}B_{PTO}\omega^2|X_i|^2. \tag{5}$$

For the whole array, the mean power is simply the sum of the mean power from each of the individual devices. For irregular waves, characterized by a wave energy spectrum S (we use a standard Jonswap spectrum with frequency spreading parameter $\gamma=3.3$), the mean power extracted is

$$P_i(H_s,T_p)=\int_0^\infty S(H_s,T_p,\omega)p_i(\omega)\,d\omega \tag{6}$$

where H_s is the significant wave height and T_p is the peak period. The yearly average power of a body i, given the probability of