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Simplified formula of hydrodynamic pressure on circular bridge piers in the time domain

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ABSTRACT

The hydrodynamic pressure expression based on radiation theory is very complicated and temporal convolution in the time domain that lead to their values difficult to be calculated. The Morison equation is a semi-empirical formula and its inertia coefficient and drag coefficient are difficult to be determined accurately. Therefore, the simplified formula based on radiation theory which is concise, accurate and global decoupling in the time domain is presented by introducing three dimensionless parameters including frequency ratio, wide depth ratio and relative height of bridge pier. By analyzing the analytical formula of hydrodynamic pressure detailed, the hydrodynamic pressure formula in the frequency domain is simplified by segmenting. The simplified formula which is global decoupling in the time domain can be expressed in the form of added mass in low frequency vibration and can be expressed in the form of added mass and added damping in high frequency vibration. The calculation results show that the simplified method is in good agreement with the analytical formula. Compared with the Morison equation whose coefficients need to be selected by experience, the coefficients of the simplified formula are only related to the relative size of bridge piers.

1. Introduction

With the development of economy and transportation, lots of deep-water bridge piers have been built around the world in recent years, and the depth of the submerged part of some bridge piers can even exceed 100 m. These deep-water bridges are always under threat of earthquakes and hydrodynamic pressures because most of them are located in zones characterized by high seismic hazard levels. As we all know, dynamic properties and dynamic response of deep-water bridges may be affected by hydrodynamic pressure. Thus, fully understanding and researching of hydrodynamic pressure of bridge piers under earthquake excitation is of great importance for the seismic design of newly-built deep-water bridges or the seismic safety assessment of old bridges. At present, there are mainly two approaches to calculate hydrodynamic pressure of bridge piers, including the Morison equation and the method based on radiation wave theory (RWM).

The Morison equation was originally used to estimate the wave force acting on offshore structures such as oil platforms, bridge piers, submerged floating tunnels and other offshore structures (Sarpkaya, 1986; Gudmestad and Moe, 1996; Manwell et al., 2007; Hiroshi, 2010). This method assumed that the characteristics of the cylinder had no significant effect on wave motion, and considered that the wave force was made up of two components, namely: a drag force proportional to the square of the velocity and a virtual mass force proportional to the horizontal component of the accelerative force exerted on the mass of water displaced by the cylinder. The inertia force is of the functional form as found in potential flow theory, while the drag force has the form as found for a body placed in a steady flow. These two force components were simply added to describe the force in an oscillatory flow by Morison et al. (1950). It can be seen that the Morison equation is not strictly from the theory of fluid mechanics. Besides, the Morison equation is mainly suitable for a slender structure (cylinder $D/L<0.2$, D denotes diameter and L means wavelength). The Morison equation contains two empirical hydrodynamic coefficients including an inertia coefficient and a drag coefficient, which are determined from experimental data. According to dimensional analysis and in experiments by Sarpkaya (1976), these coefficients depend in general on the Keulegan–Carpenter number, Reynolds number and surface roughness. The Morison equation was modified by Penzien and Kaul (1972) to analyze the hydrodynamic pressure. Later the modified Morison equation was widely used to evaluate hydrodynamic force and produced lots of prominent research works (Gao and Zhu, 2006; Li et al., 2008; Li and Song, 2010). Yang and Li (2013) presented the expanded

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Morison equation which can afford hydrodynamic pressure caused by inner water and outer water simultaneously because the standard Morison equation is unable to calculate hydrodynamic pressure induced by inner water of hollow columns under earthquakes.

The method based on radiation wave theory (RWM) was first introduced by Liaw and Chopra (1974), which was an analytical formula. There are two very important factors including water compressibility and surface waves for RWM. Studies conducted by Liaw and Chopra (1974) and Tanaka and Hudspeth (1988) and Huang and Li (2011) indicated that surface waves would be of little consequence for the dynamic response of a cylinder surround by water in earthquakes, and water compressibility could be ignored for the hydrodynamic pressure of a small diameter cylinder but had significant effect for the hydrodynamic pressure of a large diameter cylinder in the main range of seismic frequencies. If surface wave and water compressibility were ignored, the effect of the hydrodynamic pressure was similar to an 'added mass'. The radiation wave theory is widely used by lots of researchers in recent decades and many achievements have been obtained. Lai et al. (2004) studied the hydrodynamic pressure due to outer water, while Liu et al. (2008) studied the hydrodynamic pressure due to inner water and presented hydrodynamic pressure expressions for cylindrical hollow bridge piers. Wu et al. (2006) investigated the diffraction and radiation problems for two cylinders in water with finite depth and the analytical expressions of the potentials are obtained. However, these analytical expressions are so complicated that they are difficult to be utilized in practical application. Therefore, it is meaningful work to simply those complicated expressions. The analytical expressions including hydrodynamic pressure caused by outer and inner water for circular hollow piers in deep water under an earthquake were modified and simplified by Li and Yang (2013) in some extent. This approach for the calculation of hydrodynamic pressure improves the adaptability. However, compared with the Morison equation, the formula obtained by the method is complicated and it ignores the effect of water compressibility. Here in the present study, considering the effect of water compressibility, a simplified formula was developed with a higher accuracy than the Morison equation.

It can be clearly seen that the Morison equation and the RWM both have their own defect. The Morison equation is practical but it is a semi-empirical formula and low precision for fat piers. Moreover, the inertia coefficient and drag coefficient in the

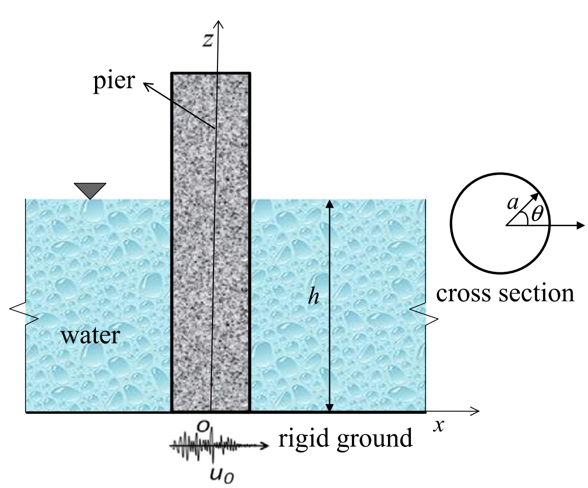

Fig. 1. Analytical model of the cylinder structure-water system under earthquake excitation.

Morison equation are difficult to determine accurately. The RWM is an analytical solution, but its expression is very complicated and it is temporal convolution in the time domain and only applicable to cylinder. Because hydrodynamic pressure should be taken as an external load to analyze the dynamic response of the whole bridge or the bridge pier concisely, the hydrodynamic pressure expression is better to be the global decoupling in the time domain. Therefore, developing a simplified formula which is concise, accurate and global decoupling in the time domain is the goal in this work.

In this study, three dimensionless parameters which are frequency ratio, wide depth ratio and relative height of the bridge pier are introduced to analyze the analytical solution based on radiation wave theory. By analyzing, the hydrodynamic pressure expression can be divided into two parts including added mass and added damping. The added damping is caused by radiation damping, which exists only when water compressibility is considered and the frequency ratio is larger than one. Simplified expressions of added mass and added damping are obtained by curve fitting. Comparing the simplified formula with the analytical solution and the Morison equation, the results show that the simplified formula is accurate and concise enough for engineering applications.

2. Governing equation and solution

2.1. Assumptions and governing equation

The bridge pier is simplified as a rigid cylinder with the bottom fixed at the rigid ground. The water in the present study is assumed to be linearly compressible and inviscid, and it undergoes a small-amplitude irrotational motion without considering the effects of surface waves. The system of the interaction between the bridge pier and surrounding water is depicted in Fig. 1. The radius of the pier is a, and the depth of water is h. The water is treated as calm before the action of earthquake and only the radiation wave would be simulated under earthquake. And the linear wave theory can be utilized because the amplitude of the wave is relatively small. A harmonic wave with frequency ω is considered here as the seismic excitation which propagates along the x-axis, as shown in Fig. 1. And displacement $u(t)$ of ground motion, therefore, can be expressed as $u(t)=U_0e^{i\omega t}$. Here, U_0 is the amplitude, t is time. The coordinate system is presented in Fig. 1.

The governing equation for the water can be written in terms of its hydrodynamic pressure in cylindrical co-ordinates, which should satisfy the Laplace's equation, that is

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2}+\frac{\partial^2 p}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{1}$$

where $p(r,\theta,z,t)$ is the hydrodynamic pressure and c is velocity of sound in water = 1438 m/s. The boundary conditions on $p(r,\theta,z,t)$ are

$$p=0 \text{ on } z=h \tag{2a}$$

$$\frac{\partial p}{\partial z}=0 \text{ on } z=0 \tag{2b}$$

$$\frac{\partial p}{\partial r}=-\rho\frac{\partial^2 u}{\partial t^2}\cos\theta \text{ on } r=a \tag{2c}$$

In addition, Sommerfeld radiation condition should be considered, as Eq. (2d).

$$\lim_{x\to\infty}\sqrt{r}\left(\frac{\partial\phi_R}{\partial r}-ik\phi_R\right)=0 \tag{2d}$$