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Numerical studies on sloshing in rectangular tanks using a tree-based adaptive solver and experimental validation

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ABSTRACT

As liquid cargo transport developing, hull structural stability and strength are highlighted in various engineering areas due to tanks partially filled with fluid. Sloshing may be accounted as unpredictable force imposed on the whole faces of the tank walls. Water movement in rectangular tanks, under rolling and horizontal excitations, is investigated by numerical and experimental approaches. A parallel code, which directly discretizes the incompressible Navier–Stokes equations, coupled with VOF and tree-based adaptive algorithm, is employed to simulate the behavior of 2D fluid motion. A series of experiments are carried out to measure the pressures on the tank walls and under the water. A good agreement is shown in this paper and the values obtained by computations are validated. Through this, some studies are done on the basis of different excitation frequencies and filled levels.

1. Introduction

Sloshing is considered as a motion of water in partially filled tanks under an external excitation, which usually represents a violent phenomenon with high amplitudes of water surface and strong localized impacts on the tank walls when the excitation frequency is close to the natural frequency of this system. This phenomenon is particularly evident under relatively low water depth. The critical depth to length ratio is 0.3368 (Faltinsen and Timokha, 2009). Sloshing is also regarded as water wave movement of the free surface which is normally sorted into different types according to their shapes: standing wave, traveling wave, hydraulic jump, breaking wave and the combination of themselves. When the resonance occurs, it is difficult to separate them from each other, because of the strong nonlinear and arbitrary phenomenon experienced by the surface. Moreover, the instantaneous loads can be significantly high which may cause structural damage in turn and even engender sufficient torque to impede the stability of the vehicle. To that end this paper considers that under relatively low water depth how the surface shape changes according to different external frequencies and what extent are the loads predicted by numerical means against experimental method.

To assess sloshing loads and the free-surface elevation, theoretical approaches based on linear and nonlinear potential flow theory and laboratory experimentation based on scaled models have been employed by many researchers. Although the sloshing technology developed for space applications is not directly applied to ship cargo tanks, Abramson (1966) studied the sloshing problem systematically, in terms of the space industries, by potential flow theory with different shapes of fuel tanks for launch vehicles. A series of analytical solutions were derived based on the linear method, and corresponding experiments were conducted. Faltinsen (1974), Faltinsen (1978) and Faltinsen (2000) applied potential flow theory of an incompressible liquid and developed nonlinear multimodal method for sloshing, and some classic formulae were obtained, which have been widely used today (Waterhouse, 1994), but only restricted to cases in simple shape tanks with the free surface elevation of a single-valued function. Akyildiz and Ünal (2005) and Akyildiz and Ünal (2006) designed an experiment which was set up to observe the behavior of liquid sloshing in rectangular container subjected to external excitation with a relatively deep water depth. Rafiee et al. (2011) studied experimentally sloshing phenomena in a thin rectangular tank (with width-to-length ratio of 0.0769) under a sway excitation, and only shallow water depth was considered.

Since the advent of computers, people started to solve fluid mechanics problem by using computational techniques. Numerical simulations have become important and widely adopted technique to deal with strong nonlinear sloshing. Wu et al. (1998) analyzed the sloshing waves using finite element method (FEM). Shallow water theory is an important mathematical model when hydraulic jump occurs, which was employed by Nakayama and Washizu (1981) to present a nonlinear analysis of inviscid and incompressible fluid motion in a tank subjected to forced oscillation. At the same time,

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both FEM and boundary element method (BEM) were combined in. Besides the aforementioned techniques, approaches based on Navier–Stokes equations have been applied to study liquid sloshing. Reynolds averaged Navier–Stokes (RANS) equations were compared with shallow water theory by Armenio and La Rocca (1996). Moreover, Liu and Lin (2009) studied sloshing using volume of fluid (VOF) method coupled with turbulent modeling. Xue and Lin (2011) similarly analyzed the surface elevation and velocity field of liquid sloshing by spatially averaged Navier–Stokes (SANS) equations coupled with virtual boundary force (VBF) method. The smoothed particle hydrodynamics (SPH) method featured mesh-free has been also applied to the study of sloshing flow. Rafiee et al. (2011) and Chen et al. (2013) both developed improved SPH techniques to simulate the sloshing flow by using more accurate integration and more applicable treatment of the boundary conditions.

However, to our knowledge, adaptive algorithm used in this field is rarely seen. Liang and Borthwick (2009) validated a Godunov-type shallow flow solver on adaptive quadtree grids by simulating linear sloshing motions in a vessel with a parabolic bed. A parallel program Gerris Flow Solver with VOF method based on adaptive quadtree grids was created by Popinet (2003), Popinet (2009) and Popinet (2010). A condensed introduction of this open-source code is summarized in Section 3.

The motivation of this paper is to demonstrate that Gerris can indeed simulate the violent fluid motion of sloshing and predict the accurate pressure. Therefore, both numerical simulation and laboratory experiment were carried out to study sloshing problems. We focus on giving comparisons of loads between the computations and laboratory experiments under the filling ratios falling into the vicinity of the critical depth mentioned in the first paragraph. For the purpose of that, we designed a series of tests of water sloshing in a rectangular tank. Because of the characteristic of the tank with a square bottom, the 3D effects cannot be ignored during our analysis. Thus, the free surface wave shape is another important aspect of this study.

2. Mathematical model

The fluid motion of sloshing can be described by the incompressible continuity equation and Navier–Stokes equations:

$$\rho\left(\frac{\partial u}{\partial t}+u\cdot\nabla u\right)=\rho F-\nabla p+\nabla\cdot(2\mu D) \tag{1}$$

$$\nabla\cdot u=0 \tag{2}$$

where, u, p, ρ are velocity, pressure, density, respectively. D the deformation tensor defined as $D_{ij}=(\partial_i u_j+\partial_j u_i)/2$. ρF the external forces in 2D can be derived and written in the moving coordinate system (Celebi and Akyildiz, 2002), as shown in Fig. 1, as follows:

$$F_1=-g\ \sin\ \theta+x_2\ddot{\theta}+x_1\dot{\theta}^2+2\dot{\theta}u_2-\ddot{A} \tag{3}$$

$$F_2=-g\ \cos\ \theta-x_1\ddot{\theta}+x_2\dot{\theta}^2-2\dot{\theta}u_1 \tag{4}$$

where A and θ are the translational and rotational amplitude of the non-inertial frame with regard to the considered time, respectively,

$$A=A_0\ \sin(\omega_s t+\varepsilon_s) \tag{5}$$

$$\theta=\theta_0\ \sin(\omega_p t+\varepsilon_p) \tag{6}$$

where, $A_0,\theta_0,\omega_s,\omega_p,\varepsilon_s,\varepsilon_p$ are the translational and rotational amplitudes, frequencies and phase differences, respectively.

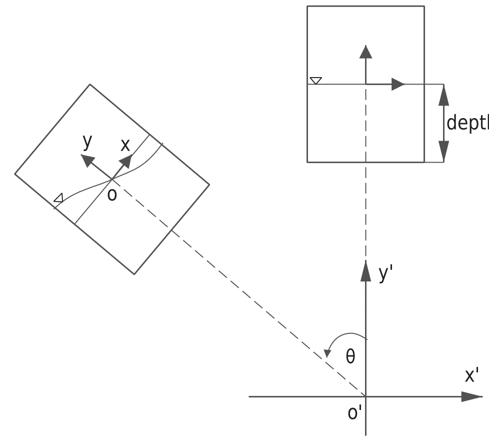

Fig. 1. Definition sketch.

3. Numerical computation

The numerical method used in Gerris has been described in detail in Popinet (2003), where a finite volume method (FVM) is applied to solve the governing equations. VOF approach is employed to capture the free-surface and to reconstruct it, and the spatial discretization of the computational domain relies on the quadtree (in 2D) adaptive method based on square grids hierarchically. Second order up-wind schemes is used to address the convective term (Bell et al., 1989). Projection and multi-grid method are used to solve the pressure Poisson equation, respectively. In the following we will summarize the main characteristics of the technique.

3.1. Spatial discretization

Square finite volumes, organized hierarchically as a quadtree, are used to discretize the domain spatially. Fig. 2 indicates an example of spatial discretization. Since the quadtree structure is applied, the adaptive algorithm is rather simple to process.

It is permitted to have different variables for the refinement criterion, for example, the norm of the local vorticity vector. A cell can be refined whenever

$$\frac{h\|\nabla\times U\|}{\max\|U\|}>\tau \tag{7}$$

where, the size of a grid is represented by the length h, and τ is the threshold value, which meets $0<\tau<1$. Then, a simple criterion based on vorticity is built here.

Compared with structural uniform grids, the adaptive algorithm can reduce time and memory space needed by computation significantly. The coarsening step is processed every time the high resolution is not necessary, and the refinement step is processed whenever the high gradient of velocity or the strong contortion rate of the free-surface occurs, since it is imperative to enhance the computational precision to guarantee the efficiency and accuracy.

The attribute of square grids is inherited by the quadtree based algorithm. This square shape of finite volume cells allows values at any points to be interpolated and allows the free-surface reconstructing conveniently, and higher accuracy is guaranteed by this as well.

Fig. 2 shows the development of the adaptive grids in the domain of one computational result (water depth = 30 cm, frequency = $1.05\omega_0$) in this paper. From this it can be seen that