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Time domain modeling of a dynamic impact oscillator under wave excitations

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ABSTRACT

This paper establishes a methodology for analyzing the dynamics of a wave-induced impact model, with emphasis on the modeling of float-over installations. The time domain model described by the Cummins equation provides an attractive way of analyzing the dynamics of marine structures with nonlinear effects. By replacing the time-consuming convolution terms, the resulting model is very efficient in dealing with nonlinear problems. The established time domain model is applied to investigate Leg Mating Unit (LMU) impacts during a float-over operation by considering the heaving motions of the whole system. Both a one-body system (considering that barge and deck move as one rigid body) and a two-body system (barge and deck moving separately) are considered in this paper. The techniques of impact maps, Poincaré maps, bifurcation diagrams and phase portraits are used to investigate the motion characteristics of the barge-deck system undergoing vertical impacts with the substructure.

1. Introduction

Marine operations generally involve nonlinearities such as a nonlinear mooring system, wave-induced impacts during a float-over installation, and viscous forces. Due to these nonlinearities, the dynamics of marine structures may exhibit sub-harmonic motion and chaotic motion, under environmental loads. One typical example is the dynamics of an articulated tower, that has been modeled as a forced bilinear oscillator with a discontinuity in the stiffness of the system due to slackening of the mooring cables, see, for example, Thompson et al. (1983, 1984) and Gottlieb et al. (1992). Another example is Virgin and Bishop's (1988) investigation of the complex dynamics of a floating semi-submersible with nonlinear mooring system in the time domain, based on a nonlinear oscillator model with a constant damping for varying periods. They analyzed the phenomena of period-doubling bifurcations leading to chaos. However, the effects of the frequency dependent radiation damping were not well addressed in any of these time domain models, since only a constant damping term was used.

Another typical nonlinear marine system is the model of wave-induced impact arising in the installation of an offshore platform by the float-over method. This method is gaining popularity for installing a large deck onto an offshore platform, due to its relatively lower operational cost and higher installation capacity (Tahar et al., 2006). By this method, after being constructed at the shipyard the deck is loaded onto a transportation barge supported on the Deck Support Frame (DSF) using Deck Support Units (DSUs), and then towed to the installation site. The deck is then transferred onto the pre-installed substructure by controlling the ballasting of the barge when the prevailing sea state is suitable. In the load transfer stage, when the system is excited by waves, the mating cones attached on the deck legs will make intermittent impact with the receptors at the top of the substructure legs. These impact loads are dissipated by the Leg Mating Units (LMUs) that are pre-installed within the receptors. In order to ensure an efficient and safe deck installation, it is critical to predict the resulting forces and dynamics of the barge-deck system with high confidence.

The nonlinear dynamics of wave-induced LMU impacts cannot be evaluated by the conventional frequency domain model based on a linear potential flow approach (Chen et al., 2012). Thus, time domain modeling of the wave-induced LMU impacts is pursued here. The Cummins (1962) equation provides an attractive way of analyzing the dynamics of marine structures in the time domain. The equation leads to a linear time invariant framework relating the motion of the structure to the wave excitations on it. By replacing the time-consuming convolution term with a state-space model, one obtains a constant parameter time domain model based on the frequency domain results. Much literature has been devoted to the

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replacement of the convolutions by a state-space model—see, for example, Yu and Falnes (1995), Kristiansen et al. (2005) and Taghipour et al. (2008). Compared to the frequency domain model, a time domain model has the advantage of analyzing nonlinear problems, providing the hydrodynamics remain linear. Any nonlinear effects can be added into the model as additional loads (Wu and Moan, 1996). In the authors' previous paper (Chen et al., 2012), the time domain model described by the Cummins equation focused on investigating the complex dynamics of a floating barge with cubic springs in surge.

This paper develops a simple wave-induced impact model for float-over operations by considering only the heaving motion of the barge-deck system, and incorporating additional piecewise linear terms in the Cummins equation. The simplified single degree of freedom (SDOF) wave-induced impact model is similar to a bilinear impact oscillator that has been extensively studied since Akashi (1958) analyzed the motion of an electrical bell. Studies of both hard and soft impacts can be found in the literature. Thompson and Ghaffari (1982) investigated the chaotic behaviour and sub-harmonic motions of a hard impact oscillator. Shaw and Holmes (1983) investigated the dynamics of a periodically forced piecewise linear oscillator by both analytical solutions and digital simulations, and provided the fundamentals of a hard impact oscillator. Lee (2005) investigated the dynamics of a hard impact oscillator based on the Runge–Kutta integration algorithm. Andreaus et al. (2010) simulated the dynamics of a bilinear soft impact oscillator based on the generalized Jacobian Matrix method. In the present paper, a combination of hydrodynamic analysis of the wave-induced dynamics, and dynamic analysis of an impact oscillator, is applied to investigate the impact behaviour of the barge-deck system under wave excitation. Both a one-body system (barge and deck moving as one body) and a two-body system (barge and deck moving separately) are considered. The numerical scheme described by Lee (2005) and Virgin and Bishop (1988) is applied here in deriving the time domain results. Analysis tools such as Poincaré maps, impact maps, bifurcation diagrams and phase trajectories are used to identify the motion characteristics.

This paper mainly has two objectives. The first is to demonstrate the methodology of combining hydrodynamic analysis and dynamic analysis to investigate the wave-induced impacts arising in a float-over installation. The second objective is to investigate how the impact behaviour changes as the wave excitations change, and perform a parametric study to analyze the sensitivity to certain control parameters of the overall system. The paper is arranged as follows. Section 2 briefly describes the theoretical background of the wave-induced impact model based on the Cummins equation. Analysis tools for identifying nonlinear dynamics are briefly reviewed in Section 3. Section 4 gives validations of the established time domain model. The hydrodynamic analysis of the float-over system is given in Section 5. Results and discussions of the wave-induced LMU impacts are illustrated in Section 6, for both one-body and two-body systems, and finally some concluding remarks are given in Section 7.

2. Theoretical background of wave-induced impact oscillator

2.1. Parametric time domain model based on the Cummins equation

2.1.1. Cummins equation and Ogilvie relations

The forces and responses of floating structures are usually obtained by means of a linear analysis in the frequency domain. Cummins (1962) derived a linear time domain equation to describe the wave excited dynamics of a floating marine structure, which is now known as the Cummins equation. For the case of zero forward speed, the equation for a floating rigid body has the following form

$$[M+A(\infty)]\ddot{x}(t)+\int_0^t h(t-\tau)\dot{x}(\tau)d\tau+Kx(t)=f^{exc}(t) \tag{1}$$

where, $\boldsymbol{x}$ is the vector of the six degrees of freedom; $\boldsymbol{M}$ is the 6×6 mass matrix; $\boldsymbol{A}$ (∞) is a constant positive-definite matrix that is known as the infinite-frequency added mass matrix; the kernel of the convolution term $\boldsymbol{h}(t)$, linked to memory effects, is the matrix of retardation functions (impulse response functions); $\boldsymbol{K}$ is the hydrostatic stiffness matrix; and $\boldsymbol{f}^{exc}(t)$ are the wave excitation forces and moments.

Ogilvie (1964) considered Eq. (1) in the frequency domain by using the Fourier transform and derived the following frequency domain model:

$$\hat{\boldsymbol{x}}(j\omega)\left\{-\omega^2[\boldsymbol{M}+\boldsymbol{A}(\omega)]+j\omega\boldsymbol{B}(\omega)+\boldsymbol{K}\right\}=\hat{\boldsymbol{f}}^{exc}(j\omega) \tag{2}$$

where $\hat{\boldsymbol{x}}(j\omega)$ and $\hat{\boldsymbol{f}}^{exc}(j\omega)$ are Fourier transforms of $\boldsymbol{x}(t)$ and $\boldsymbol{f}^{exc}(t)$. The hydrodynamic coefficients (added mass $\boldsymbol{A}(\omega)$ and radiation damping $\boldsymbol{B}(\omega)$) and wave excitation forces $\hat{\boldsymbol{f}}^{exc}(j\omega)$ can be readily obtained from 3d hydrodynamic codes such as the program DIFFRACT (Eatock Taylor and Chau, 1992; Zang et al., 2005; Walker et al., 2008; Sun et al., 2012) used in this paper. The relationship between the parameters of Eq. (1) and those of Eq. (2) were given by Ogilvie (1964)

$$A(\omega)=A(\infty)-\frac{1}{\omega}\int_0^\infty h(\tau)\sin(\omega\tau)d\tau, \tag{3a}$$

$$B(\omega)=\int_0^\infty h(\tau)\cos(\omega\tau)d\tau. \tag{3b}$$

By taking the Inverse Fourier Transform of the above equations, the impulse response function can be formulated as follows:

$$h(t)=-\frac{2}{\pi}\int_0^\infty \omega[A(\omega)-A(\infty)]\sin(\omega t)d\omega, \tag{4a}$$

or

$$h(t)=\frac{2}{\pi}\int_0^\infty B(\omega)\cos(\omega t)d\omega. \tag{4b}$$

Eq. (4b) is the preferred way to calculate $\boldsymbol{h}(t)$ since it converges faster than Eq. (4a). This then provides the way to evaluate the time domain model based on frequency domain results. In practice, due to the limitations of computation time and panel sizes, diffraction codes can only produce accurate results up to a certain frequency, say σ. In order to obtain accurate results at high frequencies, however, the size of panels needs to be very fine, resulting in a very large number of computations (Pérez and Fossen, 2008a). Therefore, the calculation of the impulse response functions $\boldsymbol{h}(t)$ is generally undertaken in the following way

$$h(t)=\frac{2}{\pi}\int_0^\sigma B(\omega)\cos(\omega t)d\omega+\frac{2}{\pi}\int_\sigma^\infty B_a(\omega)\cos(\omega t)d\omega \tag{5}$$

where, $\boldsymbol{B_a}(\omega)$ represents an asymptotic approximation of $\boldsymbol{B}(\omega)$ at high frequencies, which can be determined through polynomial fitting (Greenhow, 1986). The errors associated with this fit have been investigated by Chen et al. (2012), and it was shown how an appropriate value of the cut-off frequency σ can be determined.

2.1.2. Parametric model of the convolutions by using a state-space model (SSM)

The convolution term in the Cummins equation describes a causal linear time-invariant system (Yu and Falnes, 1995). This model is, however, cumbersome for numerical simulation and not well suited for the design and analysis of motion control systems (Kristiansen et al., 2005). In addition, it may be very time-consuming to directly evaluate the convolutions depending on