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Wave transformation by a dredge excavation pit for waves from shallow water to deep water

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ABSTRACT

In this paper, an analytic study on wave scattering by a dredge excavation pit is conducted, where the pit is assumed to be axi-symmetric and idealized with its depth profile in radial distance being a simple power function. By using techniques of variable separation and series expansion, a series solution to the explicit modified mild-slope equation (EMMSE) is constructed and the convergence condition is clarified. It is clear that this analytic solution is valid in the whole wave spectrum from shallow-water waves to deep-water waves due to the use of the EMMSE, which is superior to previous long-wave analytic solutions. Based on this analytic solution, influences of the pit dimension including its depth, bottom radius and opening radius on relative wave amplitude (i.e., the wave amplitude relative to the incident amplitude) are investigated. It is shown that the pit size plays a more important role than the sidewall steepness does in affecting the wave pattern. With the broader validity than that of the long-wave model, it is found that the wave attenuation within and in the lee side of the pit intensifies first and then decreases as the incident waves becomes short.

1. Introduction

Large volumes of sediment for land reclamation, community infrastructure protection and beach nourishment are usually obtained by offshore dredging, which leaves excavation pits. These modifications of the nearshore topography have a potential effect on wave transformation processes and thus may affect the eco-system such as benthic organisms inhabiting the excavated area (Nairn et al., 2004). This underlies the significance of the study on wave motions around the dredge excavation pit.

The theoretic study for the problem of wave transformation by an axi-symmetric pit was initiated by Longuet-Higgins (1967), who developed an analytic solution to the Helmholtz equation for wave scattering by a circular cylindrical pit. Bender and Dean (2005) considered wave transformation by an axi-symmetric three-dimensional pit or shoal, where an approximate analytic method to the Laplace equation (i.e., the axi-symmetric 3-D step model) was presented which is valid in a region of uniform depth containing an axi-symmetric bathymetric anomaly with gradual transitions in depth allowed as a series of steps approximating arbitrary slopes. Also, an exact analytic solution to the long-wave equation (LWE) was developed which is valid for shallow-water waves and for two kinds of truncated idealized bathymetries with the variable water depth profile being a power function: $h(r)=\lambda r$ or λr^{-1}, see Bender and Dean (2005, p. 336). Suh et al. (2005) presented an analytic solution to the LWE for wave propagating over an axi-symmetric quasi-idealized paraboloidal pit with the depth profile being the second order power function plus a constant: $h(r)=h_0+\lambda r^2$. Suh et al.'s long-wave solution was extended by Jung and Suh (2008) by employing a general LWE which degenerates from the extended mild-slope equation (EMSE) and easing the restriction on bathymetry from $h(r)=h_0+\lambda r^2$ to $h(r)=h_0+\lambda r^m$ with m being an arbitrary positive integer. Niu and Yu (2011) studied wave refraction over a dredge excavation pit and an analytic solution of the problem in terms of Taylor series to the LWE was constructed where the geometry of the pit was assumed to be axi-symmetric and idealized with its depth profile in radial distance being a power function: $h(r)=\lambda r^s$ with s being an arbitrary positive real number.

To extend the validity of the analytic solutions in Suh et al. (2005) and Jung and Suh (2008) from the long-wave regime to both the intermediate- and short-wave regimes, Jung and Suh (2007) employed the conventional mild-slope equation (MSE, Berkhoff, 1972) as the governing equation of their analytic model. However, because of the great difficulty in constructing an exact analytic solution to the MSE due to the implicity of the coefficients in the MSE, the direct solution to the linear dispersion relation given by Hunt (1979) must be employed as proposed firstly by Liu et al. (2004). Therefore Jung and Suh's (2007) solution is an approximate analytic solution to the MSE. Due to the feature of Hunt's solution, this approximate analytic solution is accurate in shallow and deep waters, while it is less accurate in intermediate depth waters. Although the accuracy in intermediate depth waters can be improved by increasing Hunt's

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approximation order, see Jung and Suh (2007), it is impossible to analytically judge the convergence of the series solution if Hunt's approximation order is greater than or equal to five, see Jung et al. (2008, p. 1229).

Very recently, by introducing a new variable $K = kh = k(h(r))h(r)$ with k being the wave number related to depth h, Liu and Zhou (2014) derived an explicit form of the modified MSE (MMSE, Chamberlain and Porter, 1995) in the radial direction for wave scattering by a three-dimensional axi-symmetric bathymetries, in which all the coefficients in the new equation are explicit functions of the new variable K. This explicit form of the MMSE brings us great convenience not only to study the properties (such as singularity and regularity) of the governing equation but also to seek an analytic solution to the MMSE since the use of Hunt's approximation solution is no longer necessary, see Liu et al. (2013), Xie and Liu (2013) and Zhai et al. (2013).

In this paper, wave transformation over the dredge excavation pit studied by Niu and Yu (2011) is reconsidered. Based on the explicit form of the MMSE (Liu and Zhou, 2014), by using techniques of variable separation and series expansion, a series solution to the MMSE is constructed and the convergence condition is analytically clarified. It is clear that this solution is valid in the whole wave spectrum from shallow-water waves to deep-water waves due to the use of the MMSE, therefore it is an extension to the long-wave analytic solution (Niu and Yu, 2011). Based on this solution, influences of the pit dimension including its depth, bottom radius, opening radius and sidewall steepness on relative wave amplitude (i.e., the wave amplitude relative to the incident amplitude) are investigated. Also, influence of the wave length of incident waves from shallow-water to deep-water on wave attenuation is also investigated.

2. Solution technique

Wave scattering by the dredge excavation pit is shown in Fig. 1 and the water depth is given by

$$h(r) = \begin{cases} h_1, & 0 \le r < r_1, \\ h_0\left(\dfrac{r_0}{r}\right)^s, & r_1 \le r \le r_0, \\ h_0, & r > r_0, \end{cases} \tag{1}$$

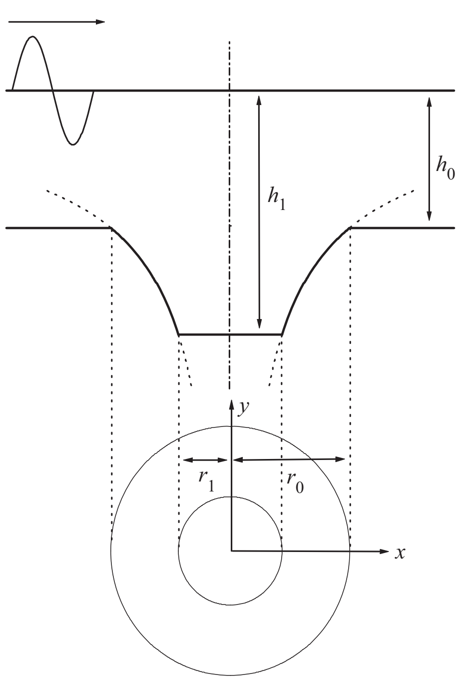

Fig. 1. Definition sketch of the physical problem.

where the power exponent s is a positive real number, $r = \sqrt{x^2+y^2}$ and $h_1 : h_0 = r_0^s : r_1^s$.

Assuming that incident waves with unit amplitude come from the left, i.e., $\eta_I(x,y) = e^{ik_0 x}$ with k_0 being the wave number related to water depth h_0 determined by the linear dispersion relation

$$\omega^2 = gk \tanh kh \tag{2}$$

with g being the gravitational acceleration and ω the angular frequency.

In two subregions with constant water depths h_0 and h_1, we can set

$$\eta(r,\theta) = \begin{cases} \sum_{n=0}^{\infty} \alpha_n J_n(k_1 r) \cos n\theta, & 0 \le r < r_1,\ 0 \le \theta < 2\pi, \\ \sum_{n=0}^{\infty} [i^n \varepsilon_n J_n(k_0 r) + \delta_n H_n^{(1)}(k_0 r)] \cos n\theta, & r \ge r_0,\ 0 \le \theta < 2\pi, \end{cases} \tag{3}$$

after the Sommerfeld far-field radiation condition of the scattered waves at the infinity is used, where i is the imaginary unit, k_1 is the wave number related to water depth h_1, $J_n(k_j r)$ is the Bessel function of the first kind of order n, $H_n^{(1)}(k_0 r)$ is the Hankel function of the first kind of order n, and the Jacobi symbol $\varepsilon_n = 1$ for $n=0$ and $\varepsilon_n = 2$ for $n > 0$, respectively. The constant coefficients α_n and δ_n are yet to be determined.

In the slope region $[r_0, r_1]$, by using the variable separation technique, $\eta(r,\theta)$ may be expressed into a Fourier-cosine series

$$\eta(r,\theta) = \sum_{n=0}^{\infty} R_n(r) \cos n\theta, \tag{4}$$

where $R_n(r)$ satisfies the following ordinary differential equation (Chamberlain and Porter, 1995)

$$\frac{d^2R_n}{dr^2} + \left(\frac{d \ln u_0}{dr} + \frac{1}{r}\right)\frac{dR_n}{dr} + \left[k^2 + \frac{u_1}{u_0}\left(\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr}\right) + \frac{u_2}{u_0}\left(\frac{dh}{dr}\right)^2 - \frac{n^2}{r^2}\right]R_n = 0, \tag{5}$$

with

$$u_0(h) = \frac{g}{2k} \tanh kh\left(1 + \frac{2kh}{\sinh 2kh}\right), \tag{6}$$

$$u_1(h) = \frac{g\ \mathrm{sech}^2 kh}{4(2kh + \sinh 2kh)}(\sinh 2kh - 2kh \cosh 2kh), \tag{7}$$

$$u_2(h) = \frac{gk\ \mathrm{sech}^2 kh}{12(2kh + \sinh 2kh)^3}[16(kh)^4 + 32(kh)^3 \sinh 2kh - 9 \sinh 2kh \sinh 4kh + 6kh(2kh + 2 \sinh 2kh)(\cosh^2 2kh - 2 \cosh 2kh + 3)]. \tag{8}$$

It is noted that, for $r \in [r_0, r_1]$, we have

$$h'(r) = -\frac{s h_0 r_0^s}{r^{s+1}}, \tag{9}$$

which is always positive in the region $[r_0, r_1]$, so $h = h(r)$ is monotonic for $r \in [r_0, r_1]$, thus has the inverse function

$$r = \psi(h) = r_0 h_0^{1/s} h^{-1/s}. \tag{10}$$

Following Liu and Zhou (2014), if we introduce the following new independent variable:

$$K = K(r) = kh = k[h(r)]h(r), \tag{11}$$

and notice that

$$h = \frac{g}{\omega^2} K \tanh K, \tag{12}$$

$$R_n(r) = R_n(\psi(h)) = R_n\left(r_0 h_0^{1/s} h^{-1/s}\right)$$