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Numerical investigation of wave radiation by a vertical cylinder using a fully nonlinear HOBEM

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ABSTRACT

A time-domain higher-order boundary element method (HOBEM) is developed to simulate fully nonlinear wave radiation by a forced oscillating structure. On the free surface, a Mixed Eulerian–Lagrangian (MEL) technique is employed in the time marching process, and mesh regridding and interpolation are applied to avoid possible numerical instability. An artificial damping layer is distributed on the outer part of the free surface to prevent wave reflection from the far-field boundary. For the calculation of wave loads, some auxiliary functions are used, instead of directly predicting the time derivative of the velocity potential. The developed model is applied to simulate a truncated vertical circular cylinder undergoing forced heave, surge or pitch motions, respectively. A series of higher-harmonic force components on the cylinder are derived by the Fourier Transformation. The added-mass and radiation-damping coefficients of the cylinder are also obtained from the least-square method. The simulated results are compared with the experimental and numerical results of other researchers. The present results are in good agreement with the experimental and other fully nonlinear results, while different with the linear and second-order solutions.

1. Introduction

Accurate prediction of nonlinear wave loads is an important topic in the design and operation of coastal and offshore structures. However, most researches that have been conducted are based on the linear or higher-order perturbation analysis. Such methods are valid only when the wave amplitude and the body motion amplitude are smaller compared with the typical body dimension and the wavelength. Beyond this condition the fully nonlinear model should be used. The major difficulties associated with the fully nonlinear method are the complicated treatment of the time-varying free surface and intersections between body and water surfaces, and the tremendous computational cost. In addition, the numerical instability is another problem to be solved. In recent years, the fully nonlinear method has been developed and applied to investigate the nonlinear wave forces in parallel with the rapid growth of numerical technique and computer power. For example, Ma et al. (2001a, 2001b), Wu and Hu (2004), and Wang et al. (2007), Wang and Wu (2010) developed a fully nonlinear numerical wave tank for simulating three-dimensional (3D) waves and wave-structure interactions by the finite element method (FEM). Alternatively, Grilli et al. (2001), Xue et al. (2001a, 2001b), Ferrant (1996,1998), Ferrant et al. (2003), and Bai and Eatock Taylor (2007) used the boundary element method (BEM) to investigate 3D overturning waves and wave-body interactions in a numerical wave tank.

The present paper focuses on the fully nonlinear wave radiation problem induced by a forced oscillating structure. Similarly, most of the previous researches on wave radiation problems are carried out based on the linear or higher-order perturbation analysis. For example, Yeung (1981) used the analytical solution to study the radiation problem of a vertical cylinder in finite-depth water. Li (1995) and Goren (1996) utilized the second-order theory to study the oscillation of a truncated cylinder in the frequency domain. A moderate time-domain simulation for the similar problem was investigated by Isaacson and Ng (1993) and Teng et al. (2002), in which the second-order perturbation was also adopted to reduce the complexity of the problem. However, the fully nonlinear simulation of the wave radiation problem is limited. Among those applications, Hu et al. (2002) and Wang et al. (2007) simulated the fully nonlinear radiated wave field using the FEM, and Bai and Eatock Taylor (2006) by the BEM. On the other hand, the experimental studies have been undertaken by Chaplin et al. (1999).

On the basic of the theory of Bai and Eatock Taylor (2006), the present paper developed a robust numerical model of surface-piercing three-dimensional bodies undergoing forced motions, such as heave, surge and pitch. The overall method is as follows. Firstly, two coordinate systems are defined to describe the position

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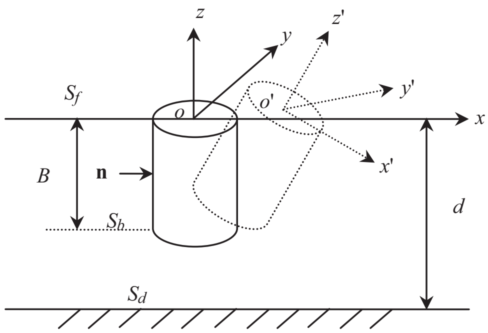

Fig. 1. The sketch of definition.

of moving bodies. The higher-order boundary element method (HOBEM) is utilized to solve the mixed boundary value problem based on an Eulerian description at each time step. One advantage of using the HOBEM is that remeshing is much easier compared with the FEM. Another advantage is that the calculation of the velocities of grid point, and the determination of the intersections between body and water surfaces, are more convenient. The 4th-order Runge–Kutta scheme and the Mixed Eulerian–Lagrangian (MEL) technique are adopted to update the free water surface boundary conditions expressed in a Lagrangian formulation. Mesh regridding and interpolation are applied on the free surface to deal with the possible numerical instability. For the calculation of wave forces, some auxiliary functions are introduced, which avoid unexpected inaccuracy and instability during directly predicting the time derivative of the velocity potential. Numerical simulations are carried out for a truncated vertical circular cylinder undergoing heave, surge or pitch motions, and the comparisons with the numerical and experimental results from other researchers are given and discussed. Moreover, a relatively comprehensive investigation of the corresponding hydrodynamic forces and wave run-up is provided.

Compared with Bai and Eatock Taylor (2006), the present paper improves the numerical implementation and analyzes further nonlinear features of wave elevations and forces. Structured quadrilateral elements are used in this paper, which have to be found to be more efficient compared with using triangular elements in Bai and Eatock Taylor (2006). A different mesh regridding and interpolation is utilized, because the method used by Bai and Eatock Taylor (2006) is only suitable for triangular elements. The mesh generation method used in this paper employs a more advanced distribution method, which is suitable for bodies of arbitrary motion. The added mass and radiation damping coefficients are decomposed by the least-square method and compared with those from the linear theory. A series of higher-harmonic force components on the cylinder are derived by Fourier Transformation and compared with the published results.

2. Mathematical formulation

A forced moving body in the open sea with the depth d is considered. A sketch of the computational domain is shown in Fig. 1. Two right-handed Cartesian coordinate systems are defined. One is a space-fixed coordinate system $oxyz$ with the origin o in the plane of the undisturbed free surface and the z-axis pointing vertically upwards. The other is a body-fixed coordinate system $o'x'y'z'$. When the body is at its equilibrium position, these two sets of coordinate systems fully coincide with each other.

The fluid is assumed to be inviscid and incompressible, and the flow is irrotational. The velocity potential $\phi\,(x, y, z, t)$ can, therefore, be introduced, which satisfies the Laplace equation

$$\nabla^2\phi = 0 \tag{1}$$

in the fluid domain Ω.

On the instantaneous free water surface S_f, the fully nonlinear kinematic and dynamic boundary conditions can be given in the following Lagrangian form (Bai and Eatock Taylor, 2009)

$$\left.\begin{array}{l}\frac{D\mathbf{X}}{Dt} = \nabla\phi \\ \frac{D\phi}{Dt} = \frac{1}{2}\nabla\phi\cdot\nabla\phi - g\eta\end{array}\right\} \quad \text{on } S_f \tag{2}$$

where g represents the acceleration due to gravity, $\mathbf{X}=(x, y, z)$ denotes the position vector of a fluid particle, η is the height of water surface, D/Dt is the full derivative defined by $(D/Dt) = (\partial/\partial t) + \mathbf{u}\cdot\nabla$ with $\mathbf{u}$ being the velocity of the fluid particle. The boundary condition on the instantaneous wetted body surface S_b is

$$\frac{\partial\phi}{\partial n} = \mathbf{V}_n \quad \text{on } S_b \tag{3}$$

where $\mathbf{V}_n$ is the normal velocity component of the body surface. It is assumed that the rotational motion amplitude is small, then the motion of a three-dimensional rigid body about its rotation center $X_r=(x_r, y_r, z_r)$ can be described in terms of six components

$$\mathbf{V}_n = (\dot{\boldsymbol{\xi}} + \dot{\boldsymbol{\alpha}} \times (\mathbf{X} - \mathbf{X}_r))\cdot \mathrm{n} \tag{4}$$

where $\mathbf{n}$ is the normal unit vector pointing out of the fluid domain in the space-fixed coordinate system, as shown in Figs. 1, $\xi=(\xi_1, \xi_2, \xi_3)$ is a vector denoting the displacements of surge, sway and heave, and $\boldsymbol{\alpha}=(\alpha_1, \alpha_2, \alpha_3)$ is a vector indicating the angles of roll, pitch and yaw about $oxyz$ measured in the clockwise direction, which can also be written as (ξ_4, ξ_5, ξ_6). In the present paper, the rotation center coincides with the mass center $\mathbf{X}_c=(x_c, y_c, z_c)$, which is placed at the origin o.

In order to avoid an abrupt start and allow a gradual development of the radiation potential, the body surface boundary condition is multiplied by the following modulation function in the simulation

$$R_m = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{\pi t}{T_m}\right)\right) & \text{if } t \le T_m \\ 1 & \text{if } t > T_m \end{cases} \tag{5}$$

where T_m is a ramp time, here chosen as twice excitation period T.

Towards the outer annulus of the circular cylindrical computational domain, an artificial damping layer is applied on the free surface so that the wave energy is gradually dissipated in the direction of wave propagation. In the present study, both ϕ- and η-type damping terms are added to the free surface conditions in Eq. (2) (Bai and Eatock Taylor, 2009).

Since the proposed problem is solved in the time domain, an initial condition must also be imposed. Here, we specify that the water surface is initially calm, so that

$$\phi = \eta = 0, \; t \le 0 \tag{6}$$

3. Higher-order boundary element method

By applying Green's second identity in the fluid domain Ω, the boundary value problem presented in the previous section can be converted in the usual manner into the following boundary integral equation

$$\alpha(p)\phi(p) = \iint_S \left[\phi(q)\frac{\partial G(q,p)}{\partial n} - G(q,p)\frac{\partial\phi(q)}{\partial n}\right] ds \tag{7}$$

where $p=(x_0, y_0, z_0)$ and $q=(x, y, z)$ are the source and the field points, respectively, $\alpha(p)$ is the solid angle coefficient, S includes the whole surface and G is the simple Green function.

For cases in which the computation domain is symmetric about the x–z plane, and the seabed is horizontal, the simple Rankine source and its image with respect to the symmetry plane $(y=0)$