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Simple absorbing layer conditions for shallow wave simulations with Smoothed Particle Hydrodynamics

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ABSTRACT

We study and implement a simple method, based on the Perfectly Matched Layer approach, to treat non reflecting boundary conditions with the Smoothed Particles Hydrodynamics numerical algorithm. The method is based on the concept of physical damping operating on a fictitious layer added to the computational domain. The method works for both 1D and 2D cases, but here we illustrate it in the case of 1D and 2D time dependent shallow waves propagating in a finite domain.

1. Introduction

The problem of non reflecting boundary conditions is an old subject of the study of wave propagation in limited domains. The so called radiation boundary conditions at infinity have been studied since 1912 by Sommerfeld, but its practical implementation in computational solutions of electromagnetic field propagation can be referred to Engquist and Majda (1977). It is obvious that the occurrence of boundaries affects the evolution of a physical event that would otherwise propagate into open space. Many different strategies have been adopted to circumvent the problem. Among numerous approaches the method of characteristics is well exploited in the fixed grid numerical methods (Poinsot and Lele, 1992). The perfectly matched layer (PML) approach, i.e. the use of an artificial absorbing layer, was devised by Berenger (1994) for simulations of electromagnetic waves and successively adopted in many wave field simulations: acoustics, seismic vibrations and fluids. The general idea of the PML approach is very simple. An absorbing layer is added to the physical domain. In this layer, sink or source terms are activated, multiplied by a coefficient varying from zero, inside the physical domain, to a maximum at the outer edge of the layer zone. The mathematical properties to be attributed to this zone can reach great accuracy and complexity, as shown in the paper by Lin et al. (2011) on recent advancements for non-linear regime of the Euler equations to be adopted in the layer. Recently Modave et al. (2010) set up a simple and accurate PML method that is useful for linear and non-linear shallow water simulations. We essentially adopt this simpler approach.

2. The absorbing layer method

In general, the model equations governing the fluid dynamics are rewritten adding a sink or source term to the original equations, as follows:

$$\frac{\partial A}{\partial t} = f\left(A, \frac{\partial A}{\partial x}, x\right) - \sigma(A - A_{out}) \tag{0.1}$$

where A is a generic fluid variable, $-\sigma(A-A_{out})$ is the corresponding sink or source term, A_{out} is the external boundary value, σ is the damping coefficient different from zero only in the damping region. With an appropriate choice of the σ spatial function this procedure produces extremely small reflection waves.

All these techniques are used for fixed grids discretization of the equations. In the Lagrangian approach the characteristic lines method has been suggested by Lastiwka et al. (2009) and Vacondio et al. (2012) which uses a simplified version of that procedure. Instead the PML approach is by far simpler, but, as far as we know, it has not been studied in the context of a Lagrangian approach. We adopted this strategy for the Lagrangian Smoothed Particle Hydrodynamics scheme and tested it in the case of waves propagating in a finite tank. We show that the results are fairly good.

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The Smoothed Hydrodynamics method (SPH) is a Lagrangian mesh-free method based on a single basic interpolating function associated with each node of the moving mesh.

Here we give the basic ideas. For an up to date detailed presentation of the SPH method see Colagrossi and Landrini (2003). A function f is interpolated from its known values at points k, by the approximation of the Dirac function integral:

$$f(x) = \int f(y)\delta(x-x')dy \Rightarrow \tilde{f}(x) = \sum_k f_k W(x,x'_k)\Delta x'_k \tag{0.2}$$

where $W(x,x'_k)$ is the interpolating function, named kernel, centered in the x'_k point. This interpolating function has a scale factor h and must have the properties to mimic the Dirac function, therefore

$$\int_{-\infty}^{+\infty} W(x/h)dx = 1 \quad \text{and} \quad \lim_{h\to 0} W(x/h) = \delta(x)$$

and $\tilde{f}(x)$ is the approximated function. Exploiting the mass density ρ, we can attribute to each moving node a mass $m_k = \rho_k \Delta x'_k$ and

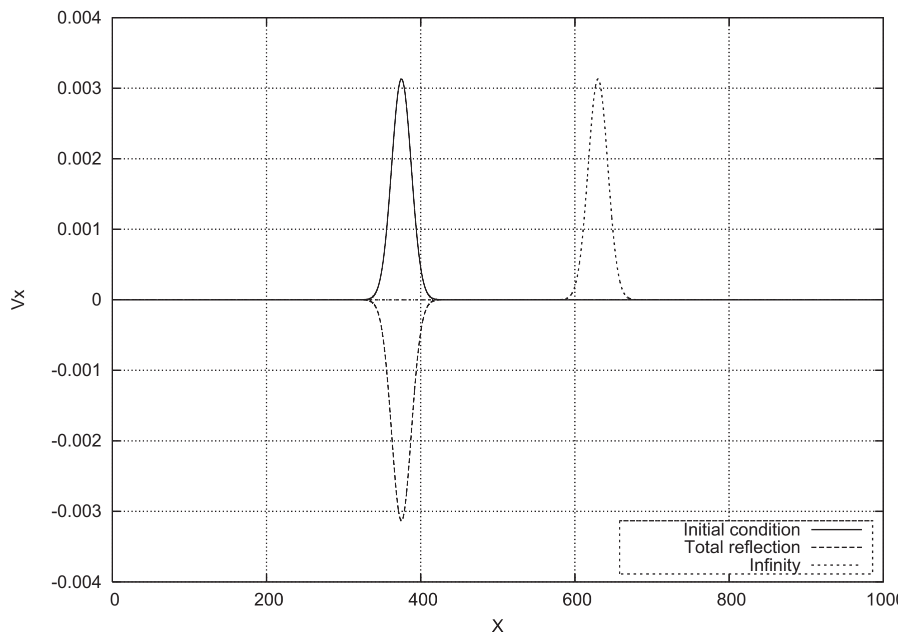

Fig. 1. Initial pulse profile chosen $H_0=1$, $x_c=3/4X$ and $A=9h$ solid line; the perfectly reflected pulse: dashed line and the infinity case: dotted line.

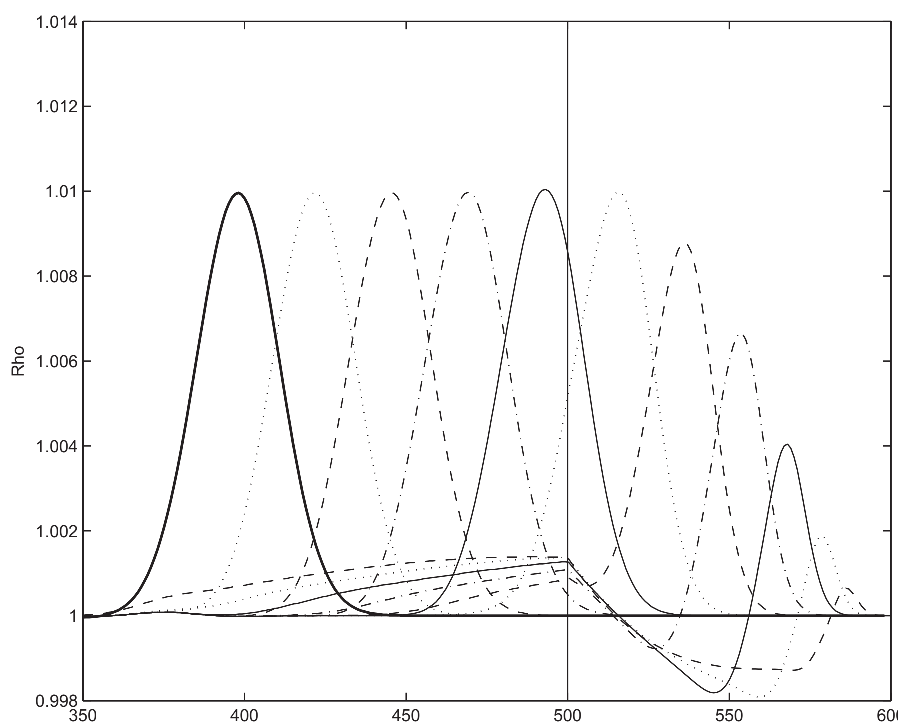

Fig. 2. Time evolution of the wave height at intervals of 7.4 s; the bold solid line represents the initial configuration.