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# The elastic critical pressure prediction of submerged cylindrical shell using wave propagation method

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ABSTRACT

A new analytical method to predict nondestructively the elastic critical pressure of a submerged cylindrical shell which is subjected to external hydrostatic pressure is presented in this paper. The structural-fluid coupling dispersion equation of the system is established considering axial and lateral hydrostatic pressure based on the wave propagation approach. The data of the natural frequencies of the system under different hydrostatic pressures is obtained by solving the coupled dispersion equation. The curve of the fundamental natural frequency squared versus hydrostatic pressure is then drawn with the data, which is straight approximately. The elastic critical hydrostatic pressure is obtained when the corresponding fundamental natural frequency decreases to zero. The results obtained from the present approach show good agreement with published results.



## 1. Introduction

The cylindrical shell is widely used in mainly underwater engineering areas such as shipbuilding, pipelines and offshore platforms. The elastic buckling loads play a very important role in the safety of these structures. The elastic buckling pressure of cylindrical shells subjected to external hydrostatic pressure has been investigated. Timoshenko and Gere (1961) gave the classical solution of the buckling pressure for a very long cylindrical shell with uniform thickness under external hydrostatic pressure by assuming that the cylindrical shell is in plane strain without considering the effect of the boundary conditions. Tatianna and Peter (1996) studied the stability of a cylindrical shell under lateral pressure in various boundary conditions based on the Flügge's shell equations. Based on the approach of energy function, the hydrostatic buckling of shells with various boundary conditions is studied by Pinna and Ronalds (2000). The study has analyzed the effect of the boundary condition on the buckling load of cylindrical shells with various lengths and concluded that the boundary condition has no effect on the buckling load for long shells.

In contrast to the destructivity of the experiments, the nondestructive prediction approach of the buckling loads has been studied by many authors (Plaut and Virgin, 1990; Singer, 1982; Souza et al., 1983; Souza and Assaid, 1991). The elastic critical loads were analyzed by fitting the curve of natural frequency and applied load of which the data was obtained from the experiments in the past years. With the development of the computer numerical simulation technology, the data can also be obtained from the simulation using FEM program. Recently the non-linear finite element software ABAQUS is widely used to study the buckling problem of cylindrical shells (Kim and Kim, 2002; Mandal and Calladine, 2000; Pinna and Ronalds, 2000; Xue and Hoo Fatt, 2002).

For submerged cylindrical shells, wave propagation approach is often used to study the vibration characteristics. Wave propagation in a cylindrical shell immersed in a fluid medium is of basic importance in fields like underwater acoustics, noise and vibration control, etc. (Junger and Feit, 1972). One of the basic studies is to solve the dispersion equation. If the traveling waves on the shell are expressed in the form of $e^{(i\omega t-ik_{ns}z)}$, where $z$ is the axial distance along the shell, $k_{ns}$ axial wavenumber and $\omega$ the frequency, the dispersion equation can be written in a general form of $F(k_{ns},\omega)=0$. The coupled natural frequency of submerged cylindrical shells can be solved from the dispersion equation (Zhang, 2002a). Wave propagation of cylindrical shells has also been investigated by many researchers in different fields (Fuller, 1981; Xu and Zhang, 1998; Wang and Lai, 2000; Zhang et al., 2001b; Zhang, 2002b; Zhang, 2002c; Zhu et al., 2007; Yan et al., 2008).

In this paper, the hydrostatic pressure is considered as an external load imposed on the cylindrical shell. Based on wave propagation method, a nondestructive approach to predict the elastic critical pressure of the submerged cylindrical shell which is assumed to be ideal and has no imperfection is investigated. The effect of the structural-fluid coupling is included in the view of wave propagation and the natural frequencies are obtained by solving the dispersion equation of the system. The elastic critical pressure is obtained by fitting the

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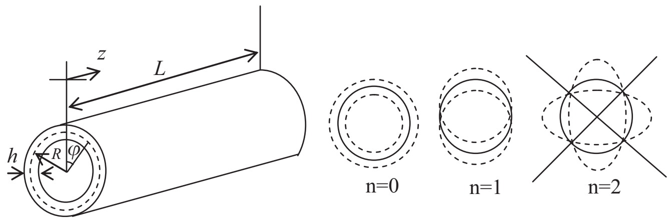


**Fig. 1.** The coordinate system and the circumferential model.

curve of the fundamental natural frequency versus the hydrostatic pressure.

## 2. Motion equations of the shell and fluid

As in Fig. 1, a thin cylindrical shell of length $L$, thickness $h$, mean radius $R$, Yong's modulus $E$, Poisson's ratio $\mu$, and density $\rho_s$, is considered to be submerged in a fluid of density $\rho_f$ where the velocity of sound is $c_f$. The cylindrical coordinates system ($r$, $\varphi$, $z$) is applied in our work to define the position of points in the region. The coordinate axis $z$ is chosen to coincide with the cylindrical shell centerline, while the coordinate axes $r$ and $\theta$ respond to the radial and circumferential directions respectively. The displacements of shell are defined by $u$, $v$, $w$ in the $z$-, $\theta$-, $r$-directions respectively.

Only harmonic motion of the coupling system is considered. Based on the classic Flügge shell equations (Flügge, 1973), the vibrational equations of cylindrical shell can be expressed as follows, in which the hydrostatic pressure is modeled as the static prestress terms in the shell equations (Keltie, 1986), (For brevity, the factor $e^{i\omega t}$ is omitted in the following expressions, $\omega$ is the circular driving frequency.):

$$[G]\begin{bmatrix}u\\v\\w\end{bmatrix}=\frac{(1-\mu^2)R^2}{Eh}\begin{bmatrix}0\\0\\P_f\end{bmatrix} \tag{1}$$

where [G] is a symmetric matrix and the elements are given by:

$$G_{11}=(1+T_1)R^2\frac{\partial^2}{\partial z^2}+\left[T_2+\frac{1-\mu}{2}(K+1)\right]\frac{\partial^2}{\partial\varphi^2}-\frac{\rho_s R^2(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2},$$

$$G_{12}=G_{21}=R\frac{1+\mu}{2}\frac{\partial^2}{\partial z\partial\varphi},$$

$$G_{13}=G_{31}=R(\mu-T_2)\frac{\partial}{\partial z}-KR^3\frac{\partial^3}{\partial z^3}+KR\frac{1-\mu}{2}\frac{\partial^3}{\partial z\partial\varphi^2},$$

$$G_{22}=R^2\left[T_1+\frac{1-\mu}{2}(3K+1)\right]\frac{\partial^2}{\partial z^2}+(1+T_2)\frac{\partial^2}{\partial\varphi^2}-\frac{\rho_s R^2(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2},$$

$$G_{23}=G_{32}=(1+T_2)\frac{\partial}{\partial\varphi}-KR^2\frac{3-\mu}{2}\frac{\partial^3}{\partial z^2\partial\varphi},$$

$$G_{33}=1+K+(2K-T_2)\frac{\partial^2}{\partial\varphi^2}+K\nabla^4-R^2T_1\frac{\partial^2}{\partial z^2}+\frac{\rho_s R^2(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2},$$

$$\nabla^4=\left(R^4\frac{\partial^4}{\partial z^4}+2R^2\frac{\partial^4}{\partial z^2\partial\varphi^2}+\frac{\partial^4}{\partial\varphi^4}\right),\ K=h^2/12R^2,$$

$$T_1=\frac{R(1-\mu^2)}{2Eh}P_0,\ T_2=\frac{R(1-\mu^2)}{Eh}P_0,$$

where $T_1$ and $T_2$ are the terms containing the effects of the hydrostatic pressure which consist of an axial prestress component and a radial prestress component. $P_0$ is the external hydrostatic pressure. $P_f$ is the fluid acoustic pressure.

The fluid around the cylindrical shell satisfies the acoustic wave equation and the equation of motion of the fluid can be written in the cylindrical coordinate system ($r$, $\varphi$, $z$) as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_f}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 P_f}{\partial\theta^2}-\frac{1}{c_f^2}\frac{\partial^2 P_f}{\partial t^2}=0 \tag{2}$$

where $P_f$ is the acoustic pressure and $c_f$ is the sound speed of the fluid. The $r$ and $\theta$ coordinates are the same as those of the shell.

## 3. Wave propagation method

According to the wave propagation approach, the displacement components of the cylindrical shell can be expressed in a traveling wave terms as (Zhang, 2002a):

$$\begin{cases}u=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}U_{ns}\cos(n\varphi)\exp(i\omega t-ik_{ns}z)\\ v=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}V_{ns}\sin(n\varphi)\exp(i\omega t-ik_{ns}z)\\ w=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}W_{ns}\cos(n\varphi)\exp(i\omega t-ik_{ns}z)\end{cases} \tag{3a–c}$$

where $U_{ns}$, $V_{ns}$, and $W_{ns}$ are the displacement amplitudes in the $z$-, $\varphi$-, $r$-axis respectively, $n$ is the circumferential mode number and subscript $s$ denotes a particular branch of the dispersion curve, $k_{ns}$ is the axial wavenumbers and $\omega$ is the angular frequency.

By solving Eq. (2), the associated form of the acoustic pressure can be expressed as:

$$P_f=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}P_{ns}\cos(n\theta)H_n^{(2)}(k_s^r r)\exp(-ik_{ns}z) \tag{4}$$

where $P_{ns}$ is the fluid acoustic pressure amplitude; $k_s^r$ and $k_{ns}$ are the radial and axial wavenumbers respectively, which have the relation $(k_s^r)^2=k_f^2-k_{ns}^2$; $k_f$ is the free wave number, $k_f=\omega/c_f$; $H_n^{(2)}$ is the $n$th Hankel function of the second order.

To ensure that the fluid remains in contact with the shell wall, the fluid radial displacement and the shell radial displacement must be equal at the interface of the shell outer wall and the fluid. This coupling condition is then:

$$\frac{\partial w}{\partial t}=-\frac{1}{i\omega\rho_f}\frac{\partial p(r,\varphi,t)}{\partial r}\bigg|_{r=R} \tag{5}$$

By substituting Eqs. (3c) and (4) into Eq. (5), the fluid acoustic pressure amplitude $P_{ns}$ can be obtained:

$$P_{ns}=[\omega^2\rho_f/\left(k_s^r H_n^{(2)'}(k_s^r R)\right)]W_{ns} \tag{6}$$

Combining Eqs. (3), (4) and (6) with Eq. (1), the structure-acoustic dispersion equation can be obtained:

$$[L]\begin{bmatrix}U_{ns} & V_{ns} & W_{ns}\end{bmatrix}^T=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T \tag{7}$$

where the elements of the matrix $[L]$ is:

$$L_{11}=\Omega^2-(1+T_1)\lambda^2-[T_2+(1+K)(1-\mu)/2]n^2,$$

$$L_{12}=-i\lambda n(1+\mu)/2,\ L_{13}=-i[(\mu-T_2)\lambda+K\lambda^3-K(1-\mu)\lambda n^2/2],$$

$$L_{22}=[T_1+(1+3K)(1-\mu)/2]\lambda^2+(1+T_2)n^2-\Omega^2,$$

$$L_{23}=(1+T_2)n+Kn\lambda^2(3-\mu)/2,\ L_{33}=1+K+K\lambda^4$$
$$+2Kn\lambda^2+Kn^4-(2K-T_2)n^2+T_1\lambda^2-\Omega^2+FL,$$

$\Omega$ is the non-dimensionless frequency, $\Omega=\omega\sqrt{\rho_s R^2(1-\mu^2)/E}$. $\lambda=k_{ns}R$, $FL$ is the fluid-loading term:

$$FL=\Omega^2\frac{\rho_f}{\rho_s}\frac{R}{h}\frac{H_n^{(2)}(k_s^r R)}{(k_s^r R)H_n^{(2)'}(k_s^r R)} \tag{8}$$

For nontrivial solution of Eq. (7), it is required that:

$$\mathrm{Det}([L])=0 \tag{9}$$

which is the dispersion equation of the coupling system.