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Development of environmental contours using Nataf distribution model

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ABSTRACT

Developing environmental contours requires knowledge of the joint probability distribution of the environmental variables; however, it is commonly the case that only the marginal distributions and the correlation structure of the environmental variables are known based on available statistical data. The Nataf distribution model provides a way to derive joint distribution models consistent with prescribed marginal distributions and a correlation structure. It has been used extensively for multivariate models of metocean variables. In this paper, the development of contours for environmental variables having joint probability density defined by the Nataf distribution model is examined. A formulation is proposed for developing environmental contours in terms of the marginal distributions of the environmental variables and their correlation coefficients. Analytical expressions are derived for the values of the environmental variables in terms of standard normal variates defining the environmental contours. The proposed method is simple to implement and provides a direct way to easily produce contours for more than two environmental variables, thus extending 2-D contours to higher dimension contour surfaces involving a greater number of environmental variables. An example shows its application to characterize multi-variate extreme sea-states using hindcast data for hurricanes and winter storms in the Gulf of Mexico.

1. Introduction

The environmental contour method was developed by Winterstein et al. (1993) in order to provide a practical way to characterize multi-variate environmental hazards at a site for structural design purposes. An environmental contour is the locus of all environmental variables that can be searched to find the maximum system response associated with a exceeding probability commonly thought of in terms of a reliability index. The environmental contour approach was derived in connection with the inverse FORM method; its main advantage lies in that the description of the environmental variables is uncoupled from the system response. It was initially applied for offshore engineering and since then has been used to derive design loads for offshore structures (Winterstein and Engebretsen, 1998; Winterstein et al., 1999; Niedzwecki et al., 1998; Haver and Winterstein, 2009). Environmental contours have also been used for earthquake and wind engineering applications (Van de Lindt and Niedzwecki, 2000; Saranyasoontorn and Manuel, 2005, 2006).

Developing environmental contours requires a joint probability description of the environmental variables. Use is made of the Rosenblatt transformation to relate the environmental variables in the physical space with independent standard normal variables in the so called standard normal space. The environmental contours for reliability β, associated with the probability of exceeding a capacity threshold, are obtained by mapping into the physical space the standard normal variates at distance β from the origin in the standard normal space. Since the Rosenblatt transformation is used, it is customary to describe the joint probability distribution function (PDF) of the environmental variables by means of a set of conditional distributions (see e.g. Winterstein and Engebretsen, 1998; Winterstein et al., 1999; Niedzwecki et al., 1998; Haver and Winterstein, 2009; Van de Lindt and Niedzwecki, 2000; Saranyasoontorn and Manuel, 2005, 2006). For instance, the joint probability description of say three variables X_1, X_2, and X_3 is given in terms of a marginal distribution for X_1 and conditional distributions for X_2 given X_1, and for X_3 given X_1 and X_2. Variable X_1 may be chosen as a main environmental variable from the view point of its relevance in the system response. In offshore engineering a marginal distribution is usually estimated for significant wave height, and then the distribution of peak period is modeled conditional on wave height. The amount of data required for estimating the parameters in the conditional distributions increases greatly with the problem dimension; thus, it may be difficult to apply these models without making some independence assumptions when

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more than three or four variables are considered. In engineering problems it is more common that the marginal distributions of the random variables and their correlation structure be estimated based on statistical data.

Under Nataf transformation, the random variables can be mapped into correlated standard normal variables provided their joint density is defined by a Gaussian copula. In such case, the joint PDF of the random variables is given by the so called Nataf distribution model which is defined in terms of the marginal distributions and correlation coefficients. Thus, the Nataf transformation provides a way to derive multivariate probability distribution models consistent with prescribed marginal distributions and a correlation structure (Liu and Der Kiureghian, 1986). In a number of studies, models for the joint probability distribution of metocean variables, involving significant wave height, wind speed, peak period, mean zero up-crossing period, and surface current velocity, have been developed based on Nataf distribution (Ditlevsen, 2002; Wist et al., 2004; Fouques et al., 2004; Sagrilo et al., 2011), showing that it is not only relatively easy to implement but also gives reasonably well fitting joint distribution models.

In this paper, we examine the development of environmental contours considering environmental variables with a Gaussian copula and joint PDF given by the Nataf distribution model. Analytical expressions are formulated for the environmental contours in terms of their marginal distributions and correlation coefficients. They are applied to 2-D and 3-D environmental contours and the effect of the correlation coefficient on the contours is examined. Numerical examples of 3-D environmental contours for extreme values of significant wave height, peak period and wind velocity using hurricane and winter storm hindcast data for the Bay of Campeche in the Gulf of Mexico are also included and discussed.

2. Environmental contours

Consider a vector of n uncertain environmental variables $\vec{X}=(X_1,X_2,\ldots,X_n)$ and the response R of a system expressed as a deterministic function of $\vec{X}$, $R=r(\vec{X})$. Let r_c denote a deterministic design capacity of the system associated with an exceeding probability thought of in terms of a reliability index β. In the inverse FORM method r_c is determined as follows (Winterstein et al., 1993):

$$r_c=\max\,\psi(\vec{U});\quad \text{subject to } |\vec{U}|=\beta \tag{1}$$

where $R=\psi(\vec{U})$, $\vec{U}=(U_1,U_2,\ldots,U_n)$ is a vector of uncorrelated standard normal variables obtained from the Rosenblatt transformation (Rosenblatt, 1952):

$$\Phi(u_1)=F_{X_1}(x_1)$$
$$\Phi(u_i)=F_{X_i|\vec{X}_{i-1}}(x_i|\vec{x}_{i-1}),\qquad i=2,\ldots,n \tag{2}$$

$F_{X_i}(x_i)$ are the marginal probability distributions and $F_{X_i|\vec{X}_{i-1}}(x_i|\vec{x}_{i-1})$ are the conditional probability distribution functions of X_i given $\vec{X}_{i-1}=(X_1=x_1,\ldots,X_{i-1}=x_{i-1})$. Note that only environmental variables are considered in the deterministic response function $R=r(\vec{X})$; uncertainties in the system capacity are neglected. In the inverse FORM approach the capacity r_c is determined searching all possible design points on the hypersphere $|\vec{U}|=\beta$, in order to find the maximum response $\psi(\vec{U})$ that the system must withstand. Thus, the independent normal variables on the hypersphere $|\vec{U}|=\beta$ can be mapped into values of vector $\vec{X}$ which define the environmental contour. An advantage of the inverse formulation is that such description of the environmental variables $\vec{X}$ is uncoupled from the response R. These environmental contours may then be used to find the system response associated with a return period. For stochastic response "inflated" environmental contours based on omission factors have been proposed (Winterstein et al., 1993).

Let p_F denote the probability of exceeding r_c and $\Phi(\cdot)$ denote the standard normal probability distribution function. Assuming that the occurrence of extreme natural events can be modeled as a Poisson process with mean annual rate λ_E, the annual exceeding probability is then

$$p_a=1-\exp(-\lambda_E p_F) \tag{3}$$

The return period of the system response associated with a reliability $\beta=-\Phi^{-1}(p_F)$ is defined as

$$T_R=\frac{1}{1-\exp(-\lambda_E\Phi(-\beta))} \tag{4}$$

and hence

$$\beta=-\Phi^{-1}\left(-\frac{\ln(1-1/T_R)}{\lambda_E}\right) \tag{5}$$

The environmental contours are then defined by all values of the environmental variable $\vec{X}=(X_1,X_2,\ldots X_n)$ in the physical space corresponding to those vectors $\vec{U}=(U_1,U_2,\ldots,U_n)$ in the Rosenblatt space such that $|\vec{U}|=\beta$, where β is associated with a return period as shown in (5) (Winterstein et al., 1993). In other words, the environmental contour is the image in the physical space of environmental variables corresponding to an n-dimensional hypersphere of radius β in the Rosenblatt space, as illustrated in Fig. 1.

3. Proposed formulation

The Nataf transformation of random vector $\vec{X}=(X_1,X_2,\ldots X_n)$ is defined as follows (Nataf, 1962):

$$y_i=\Phi^{-1}[F_{X_i}(x_i)],\quad i=1,\ldots,n \tag{6}$$

If $\vec{X}$ has a Gaussian copula, then $\vec{Y}=(Y_1,Y_2,\ldots Y_n)$ is a Gaussian vector with correlation matrix $R_Y=E[\vec{Y}\vec{Y}^T]$ and standard normal marginal distributions (Lebrun and Dutfoy, 2009). The correlation coefficients ρ_{ij} in matrix R_Y can be calculated in terms of the correlation coefficients between X_i and X_j. Approximate expressions have been proposed to compute ρ_{ij} (Liu and Der Kiureghian, 1986). Given a sample of m observed values $x_{i,k}$, $k=1,\ldots,m$ of X_i, $i=1,\ldots,n$, ρ_{ij} can be approximated by Pearson's correlation coefficient:

$$\rho_{ij}=\frac{\sum_{k=1}^{m}y_{i,k}y_{j,k}-(1/m)\sum_{k=1}^{m}y_{i,k}\sum_{k=1}^{m}y_{j,k}}{\sqrt{\sum_{k=1}^{m}y_{i,k}^2-(1/m)(\sum_{k=1}^{m}y_{i,k})^2}\sqrt{\sum_{k=1}^{m}y_{j,k}^2-(1/m)(\sum_{k=1}^{m}y_{j,k})^2}} \tag{7}$$

where $y_{i,k}$, $i=1,\ldots,n$; $k=1,\ldots,m$ are the transformed values of $x_{i,k}$ obtained from (6).

From Nataf transformation, it follows that the joint distribution function of $\vec{X}$, $F_{\vec{X}}(x_1,x_2,\ldots,x_n)=P(X_1<x_1,X_2<x_2,\ldots)$, can be expressed as

$$F_{\vec{X}}(x_1,x_2,\ldots,x_n)=P(Y_1<\Phi^{-1}(F_{X_1}(x_1)),\ldots,Y_n<\Phi^{-1}(F_{X_n}(x_n))) \tag{8}$$

The probability in the right hand side of (8) is given by the multivariate Normal distribution of $\vec{Y}$, $\Phi_n(\vec{y};R_Y)$, so that

$$F_{\vec{X}}(x_1,x_2,\ldots,x_n)=\Phi_n(\Phi^{-1}(F_{X_1}(x_1)),\ldots,\Phi^{-1}(F_{X_n}(x_n));R_Y) \tag{9}$$