Ocean Engineering 54 (2012) 46–50

Contents lists available at SciVerse ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Bodies without secondary flows in their 3D boundary layers

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ARTICLE INFO

Article history:
Received 11 December 2010
Accepted 1 July 2012
Available online 31 July 2012

Keywords:
Body shape
3D boundary layer
Secondary flow
Streamlines
Constant pressure surface

ABSTRACT

There is no secondary flow in a boundary layer where streamlines coincide with the surface geodesic curves and earlier this coincidence was mathematically proven for constant pressure surfaces in ideal incompressible fluid. Two 3D inverse problems on determination of such surfaces are solved here: Design of a surface between the given body bow and an initially undetermined body cylindrical central part; Fitting such a surface with an initially undetermined bow to the given body cylindrical part.

1. Introduction

There are two kinds of streamlines around three-dimensional bodies (like hulls of ships): streamlines over the body surface and streamlines of inviscid flow surrounding the body boundary layer. It is possible to connect these streamlines by the normal to the body surface and then find out that there is usually an angle between these streamlines. Thus, there is a flow component perpendicular to them (a secondary flow) within the boundary layers, it is illustrated in Fig. 1. Besides the energy losses associated with their generation, the secondary flow undesirable effects include a significant non-uniformity of the body wake flow.

There is a possibility to mitigate these undesirable effects. As known since the 1950s and later recalled by Schlichting and Gersten (1999), the 3D boundary layers without secondary flows can exist over a special kind of 3D surfaces. There is no secondary flow where the inviscid flow streamlines coincide with the surface geodesic curves. As was also noted by Birkhoff and Zarantonelo (1957), such coincidence is mathematically proven for surfaces with the constant pressure in steady flows of ideal incompressible fluid. There was no example of such surface/body in Birkhoff and Zarantonelo (1957), but during the following decades, various numerical methods for determination of 3D constant pressure surfaces have been developed by Street (1977), Amromin et al. (1989) and by others later. Nevertheless, there have been no known attempts to analyze the general characteristics of such surfaces and their potential applicability to engineering. For making such attempt practical, it is also important to take into account the design requests inherent, for example, to airplanes and ships: The necessity for a convenient cargo location and size limitations. Because of this, such bodies often have cylindrical middle parts. Therefore, only the body edges (the bow and stern) can be allowed for fitting their shapes to the pressure constancy condition.

This paper presents several examples of the forward edges (bow) designed with the constant pressure over their surfaces. There is no consideration of sterns because the described fitting method is based on the ideal fluid theory. Its employment for sterns with their thick boundary layers would be less confident.

1.1. Method to determine 3D constant pressure surfaces in ideal fluid

Determination of a 3D flow with a constant pressure on the surface can be carried out by solving the following boundary-value problem for the velocity potential:

$$\Delta\Phi = 0 \tag{1}$$

$$\partial\Phi/\partial N|_S = 0 \tag{2}$$

$$U^2 - 1|_{S_b} = 2(P_\infty - P_b)/\rho U^2 = \text{const} \tag{3}$$

A flow scheme is presented in Fig. 2.

The surface S^* is considered as an approach to S_b. For solving such a nonlinear problem, there are three consecutive steps in the iterative algorithm developed herein:

1. The first step is solving the linear forward problem of determining the potential flow around the known (tried) boundary that includes the known parts of the body surface and an

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http://dx.doi.org/10.1016/j.oceaneng.2012.07.013

Nomenclature

B	horizontal dimension of obstacle
C_P	dimensionless pressure coefficient
C_{Pb}	C_P value on the bow
h	distance between surface-solution and S^* counted along N^*
N	normal to S
N^*	normal to the tried surface S^*
P_∞	ambient pressure
S	combination of all boundaries of the flow
S_b	a constant-pressure part of S
q	supposedly small intensity of sources
T	a half of vertical dimension of obstacle
T^*	tangent to the meridian section of S^*
U	$\lvert \text{grad}\ (\Phi) \rvert$
U^*	value of U obtained from the solution of the forward problem
σ	$-C_{Pb}$
Φ	velocity potential
φ	perturbation of the potential Φ

approach to its undetermined part. This step involves Eqs. (1) and (2) only and can be solved by the Hess and Smith (1967) method, for example.

2. Its second step is correction of this undetermined part by solving a linear inverse potential problem. The assumption $|(C_{Pb}-C_P)/(1+C_P)| \ll 1$ for the initial surface results also into the following inequities:

$$|h/l| \ll 1; \quad |(N,N^*)-1| \ll 1 \tag{4}$$

Here parenthesis with two vectors means their scalar product. Since inequalities (4) are satisfied, the perturbation technique can be applied to the nonlinear problem with Eqs. (1) and (3). The pressure in Eq. (3) must be calculated through $\nabla\Phi\cdot\nabla\Phi=(\partial\Phi/\partial N)^2+(\partial\Phi/\partial\tau)^2+(\partial\Phi/\partial P)^2$. The potential derivatives by unknown directions can be represented through its derivatives by known directions

$$\partial\Phi/\partial N=\partial\Phi/\partial N^*(N,N^*)+\partial\Phi/\partial T^*(N,T^*)+\partial\Phi/\partial P^*(N,P^*);$$

$$\partial\Phi/\partial\tau=\partial\Phi/\partial N^*(\tau,N^*)+\partial\Phi/\partial T^*(\tau,T^*)+\partial\Phi/\partial P^*(\tau,P^*);$$

$$\partial\Phi/\partial P=\partial\Phi/\partial N^*(P,N^*)+\partial\Phi/\partial T^*(P,T^*)+\partial\Phi/\partial P^*(P,P^*).$$

Here $P=N\times\tau$, $(N-N^*,T^*)=-\partial h/\partial T$. Taking into account Eq. (4),

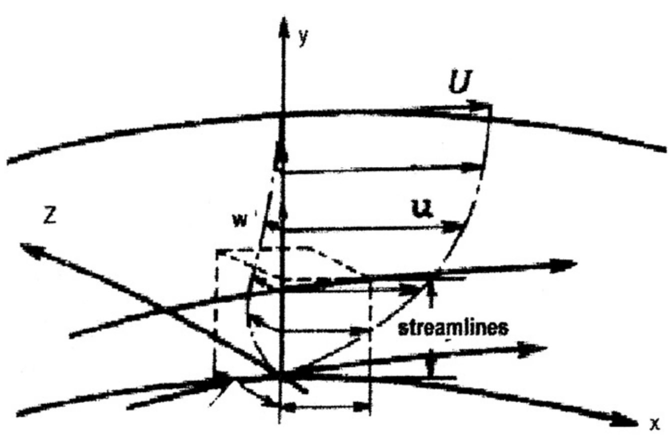

Fig. 1. Typical velocity distribution in a 3D boundary layer with a secondary flow along the z-axis.

and omitting second order terms, one can rewrite these derivatives as $\partial\Phi/\partial N=\partial\Phi/\partial N^*-\partial\Phi/\partial T^*\partial h/\partial T^*$, $\partial\Phi/\partial\tau=-\partial\Phi/\partial N^*\partial h/\partial T^*+\partial\Phi/\partial T^*$.In any iteration the potential derivatives on the corrected surface can be linearly extrapolated from its preliminary shape. Then

$$\partial\Phi/\partial N^*=\partial\Phi/\partial N^*\{S^*+\partial\varphi/\partial N^*\{S^*\}+h\partial^2\Phi/\partial N^{*2}\{S^*\};$$

$$\partial\Phi/\partial T^*=\partial\Phi/\partial T^*\{S^*\}+\partial\varphi/\partial T^*\{S^*\}+h\partial^2\Phi/\partial N^*\partial T^*\{S^*\}.$$

Taking into account Eqs. (1) and (2) on S^*, one can rewrite these derivatives on S as $\partial\Phi/\partial N=\partial\varphi/\partial N^*+U\partial h/\partial T^*$; $\partial\Phi/\partial\tau=-U+\partial\varphi/\partial T^*-hkU$. Here $\varphi\{q,S_c\}=(1/4\pi)\iint_{S_c}(q/r)dS$, $r=\sqrt{(x-x^*)^2+(y-y^*)^2+(z-z^*)^2}$, the point $\{x^*,y^*,z^*\}$ is located on S^*, κ is the curvature of S_b, Thus, used for the surface correction Eqs. (2) and (3) can be transformed into

$$\frac{\partial\varphi}{\partial N^*}=-\frac{\partial}{\partial T^*}hU^* \tag{5}$$

$$U^*\frac{\partial\varphi}{\partial T^*}-U^{*2}\left(kh+\frac{1}{2}\right)+\frac{1}{2}=-\frac{C_{Pb}}{2} \tag{6}$$

The function h found from Eqs. (5) and (6) is employed to correct S_c before the next iteration, but these equations are not applicable to a small vicinity of the obstacle, where κ cannot be assumed small. The special asymptotic (Gurevich, 1970) for h in this vicinity is employed and matched with the solution of Eqs. (5) and (6) out of this vicinity. For the simplification of computations, it is useful to keep in mind that

$$\frac{\partial\varphi}{\partial T^*}(t)=\frac{1}{2\pi}\int_0^{T_T}\frac{q(\tau)dt}{t-\tau}+\varphi^*(q,x,y,z)$$

where the first term in the right-hand side is the integral Cauchy defined on a 2D contour, but the function φ^* has no singularity. Therefore Eq. (6) is solved with the inversion (Gakhov, 1966) of this integral.The leading edge of the surface S^* must be defined with some criterion independent on Eqs. (1) and (3). This edge may be a given line, but for smooth body surfaces, the Brillouin–Villa condition is often used. This condition requires continuity of the streamline curvature (Birkhoff and Zarantonelo, 1957), but for computations, it is more convenient to use the condition consequence and lay this edge along the points of local pressure

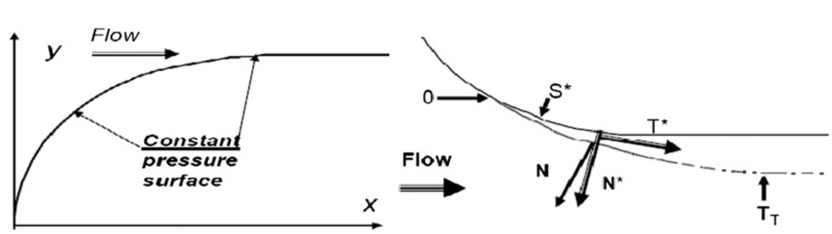

Fig. 2. Flow scheme (in the left) and comparison of its boundaries iteratively changed during solving of the boundary-value problem (1)–(3) (in the right).